ECON 7343 - Homework 2

Due Friday, Sep. 15th

1. This is a Lucas-style model of growth with human capital. Output is produced according to $y = k^{\alpha}\left[(1-u)\,h\right]^{1-\alpha}$ where $(1-u)$ is the fraction of time spent working. There is no population growth. Physical capital accumulates according to $\dot{k} = sy - \delta k$ where the savings rate is exogenous. When workers are not working, they are acquiring human capital. The only input to this is time, and the evolution is $\dot{h} = u - \delta h$. Assume that $s = \delta$ for simplicity. A) Describe the steady state of the model. Is it one with constant level of output or a constant growth rate of output per person? B) Solve for the level of u that maximizes steady state income or growth rate (depending on which one is constant in steady state). C) Let u^* be the maximizing value from B. Suppose the economy is at $u < u^*$ and then jumps to u^*. Use a phase diagram in h, k to analyze the behavior of h and k in response to this jump. Draw time series pictures showing how

2. Consider a model with two factors of production, physical and human capital. Production is $y = k^{1/2}h^{1/2}$. Equations of motion are $\dot{k} = s_k y - (n+\delta)\,k$ and $\dot{h} = s_h y - (n+\delta)\,h$. A) Will this model have steady state growth or steady state output per capita? B) Solve for the appropriate steady state. C) Suppose that initially $s_k = 0.2$ and $s_h = 0.3$. Now s_h drops to 0.25 and s_k rises to 0.25. Use a graph to show what will happen to the ratio of physical to human capital. Graph what happens to the growth rates of human capital and physical capital over time following this change in the savings rate. D) What is the effect of this change in the allocation of savings on the long run growth rate of k, h, y?

3. Consider the following variation on the Solow model. Suppose that the true production function is $y = A_1 k^{\alpha} + A_2 k$. There is no exogenous technological change. Population grows at n and capital depreciates at δ. Assume all countries in the world have identical values for A_1 and A_2 and that they all have the same savings rate. Countries differ only in their initial level of capital per person. Discuss the extent to which countries with different initial levels of capital per person will or will not converge over time. Distinguish, if appropriate, between different special cases based on the values of the parameters.

4. Consider an augmented Solow model which includes human capital. Let k be physical capital and h be human capital. Both capital stocks depreciate at the rate δ. There is no population growth. Let s_k be the fraction of output invested in physical capital and s_h be the fraction of output invested in human capital. The production function is $y = A_k k + A_h h$, where A_k, A_h are constants. Assume that $s_k A_k < \delta$ and $s_h A_h < \delta$. Analyze the dynamics of this model using a phase diagram in h/k space. Describe the paths of h and k from different initial positions. Under what conditions does the model produce "endogenous growth" and what happens when this condition is not met?

5. Consider the following model. Output is produced according to the production function $y = Ak$. There is no technological progress. Population grows at the rate n. A constant fraction, s, of output is saved. Capital depreciates in an unusual fashion: every period, d units of capital per person depreciates. Analyze the dyanmics of this economy. Describe the conditions under which there will be steady states, endogenous growth, etc.. Calculate the long-run rate of growth, if there is one.

6. Production, in per person terms, is $y = k^{\alpha} z^{1-\alpha}$, where k is physical capital and z is the stock of infrastructure (ports, roads, etc.) that is provided by the government. The two stocks accumulate as follows:

$$\frac{\Delta k}{k} = sy - (n+\delta) \qquad (1)$$

$$\frac{\Delta z}{z} = \tau y - (n+\delta) \qquad (2)$$

which implies that $(1 - s - \tau)$ of output is left for consumption.

(A) What is the steady state ratio of k/z?

(B) In steady state, what is the growth rate of output per capita?

(C) The economy is in steady state. The government then increases the tax rate τ. Draw three (3) graphs showing how y, k/z, and c evolve following the change.

7. Consider a country with a production function of $y = k^{\alpha}$. Population grows at the rate n and capital depreciates at the rate δ. There is no technological change. Consumption is each to a constant fraction of output, denoted $\bar{c}$. In addition, every period a payment in the amount of p per capita must be made to the foreign power which provides protection for this country. All output that is not consumed or paid to the foreign power is invested. A) Write down the differential equation governing the evolution of the per capita stock of capital in this country. B) Draw the Solow diagram for this country. Is there more than one equilibrium level of the capital stock and of output? Is so, identify all the equilibria. Indicate the dynamics on the diagram - that is, show to which equilibria an economy will move given it's initial stock of capital per person.