

ECON 7343 - Homework 4

Due Friday, Sep. 29th

1. Consider a regular Solow model where productivity growth is determined by

$$\frac{\Delta E_{t+1}}{E_t} = \theta R_t^\lambda E_t^{\phi-1}. \quad (1)$$

and $R_t = s_R N_t$. Population grows at rate n , and capital accumulates according to the standard Solow equation, with savings rate of s_K and depreciation of δ . Assume the economy begins in steady state, and has $0 < \lambda < 1$ and $\phi < 1$. (A) There is an exogenous shift up in s_R . Plot the path of E_t , k_t , and y_t over time following this shift. (B) Now, assume the economy has $\phi = 1$, and again let there be an exogenous shift up in s_R . Plot the time path of E_t , k_t , and y_t following this shift.

2. Start with the exact same model from problem 1, with $\phi < 1$, and in steady state. There is an exogenous jump in E_t at time zero (you can imagine that aliens land and give us access to some new technology). Plot the time path of E_t , k_t , and y_t following this one-time jump in E_t .
3. You have data on the following:

Variable	2005	2006	2007
Output (Y)	1000	1050	1020
Capital (K)	800	900	910
Labor (N)	40	44	39
Labor's share (s_n)	0.6	0.6	0.6

(A) Calculate the residual for each year. (B) Calculate E_t for each year, assuming that the markup is $\mu = 1$. (C) Calculate E_t for each year, assuming that $\mu = 2$. (D) Calculate E_t for each year, assuming that $\mu = 5$.

4. Set up the Lagrangian for a two period Fisher model with wages equal to W_1 and W_2 , a discount rate of θ , and an interest rate of r . Solve for the Euler equation relating consumption in period 1 and period 2. Does the relationship of consumption between period 1 and period 2 depend on a) the size of wages or b) the distribution of wages? Now imagine that person lives in a world with no financial system, so that they cannot save or borrow at all. They have to consume exactly their wages in each period. Solve for the interest rate that makes this the optimal outcome in the Fisher model. How does this interest rate you just solved for change with the ratio of W_2/W_1 ?
5. Consider a two-period Fisher model in which utility in each period is CRRA. Wages are equal to W in each period. Solve for the derivative of first period savings with respect to the interest rate: $\partial S/\partial r$. For what values of σ is the sign of $\partial S/\partial r$ always negative? What are the conditions under which $\partial S/\partial r$ is positive?
6. Gertrude lives for two periods. In the first period she earns a wage of 100. In the second period her wage is zero. She earns interest on her savings of $r > 0$. Her utility is $U = c^{1-\sigma}/1 - \sigma$ and she has a

zero discount rate. For what values of σ will her first period consumption be equal to 50? For what values will it be less than 50? For what values will it be exactly equal to 50?

7. Consider the optimal consumption path of Abigail, who will live exactly T periods. In the first period, she has earnings of one dollar. Subsequently, her earnings grows at the rate g in each period, so that second period wages are $(1 + g)$, third period wages are $(1 + g)^2$, and so on. Abigail can borrow and lend freely at an interest rate of zero, and she has a zero discount rate. Her utility is CRRA. She starts life with zero assets and dies with zero assets. Calculate her optimal savings in the first period of life. What is the effect of increasing g on her first period saving. Explain.
8. Take a standard consumption problem with infinity-lived individuals, where they take their income stream and interest rates as given. They have CRRA utility. The interest rate is $r = 0.05$, time preference is $\beta = 0.96$, and risk aversion is $\sigma = 2$.
 - (A) For someone with initial assets of $a_0 = 500$ and wages of $w_t = 0$ for all t , what is initial consumption, c_0 ?
 - (B) For someone with initial assets of $a_0 = 0$ and wages of $w_t = 1$ for all t , what is initial consumption, c_0 ?
 - (C) If the interest rate drops to $r = 0.02$, now what is c_0 for the period in part (A)?
 - (D) If the interest rate drops to $r = 0.02$, now what is the c_0 for the period in part (B)?