ECON 7343 - Homework 5

Due Friday, Oct. 6th

1. Consider a Ramsey model with depreciation of δ, population growth of n, a time discount rate of β and production function of $y = k^{1/2}$. Solve for the steady state level of consumption per capita in terms of the three parameters.

2. Consider two Ramsey economies which are the same in every respect except for their time discount rates, β. People in country A discount the future more than those in country B (i.e. $\beta^A < \beta^B$). Assume both countries start off at the same initial capital stock, k_0, which is below both of their steady states. Which country will have higher initial consumption? Is it possible for the two countries stable arms to cross?

3. A Ramsey model with population growth of n is in steady state. There is an unanticipated increase in n. Draw graphs of how $\ln c_t$, $\ln k_t$, and r_{t+1} change over time in response to the change in population growth.

4. This problem involves a decentralized Ramsey model. Individuals have utility of $V = \sum_{t=0}^{\infty} \beta^t u(c_t)$ and dynamic budget constraint of $a_{t+1} = (1 + r_t)a_t + w_t + x_t - c_t$. Individuals take the time path of w_t, x_t and r_t as given. They will try to optimize lifetime utility by selecting a consumption path given their initial assets a_0.

   Firms operate production technologies of $y_t = f(k_t)$, which are in per-worker terms. Firms are profit-maximizing, taking the wage rate w_t and rental cost of capital, R_t, as given. There is no population growth.

   The financial sector is not perfectly competitive. The financial sector collects a percentage, ϕ, of the total savings in the economy as their profits. These profits are returned as dividends back to the individual members of the economy in equal shares, meaning that $x_t = \phi k_t$. However, individuals do not take into account how x_t is determined when they make their optimization. They take x_t as given.

   A. Write down the Euler equation for the economy. That is, using what you know from above, write down the Euler equation showing how c_{t+1} and c_t are related to the value of k_{t+1}.

   B. Write down the economy-wide budget constraint. That is, the equation relating k_{t+1} to k_t and c_t.

   C. What is the steady state value of k in this economy?

   D. There is an unexpected, permanent shock upwards to ϕ. Draw diagrams showing how c_t, R_t, and r_t respond to the shock.

   E. What value of ϕ would make the steady-state level of k equal to the Golden Rule level of k^{GR}? Briefly explain the intuition behind this answer.

5. Take a Ramsey model in which the budget constraint is $k_{t+1} = f(k_t) + (1 - \delta)k_t - (1 + \tau)c_t$, where τ functions like a consumption tax. The economy starts in steady state in period zero. Draw a graph of $\ln c_t$ for each of the following four cases:

- In period 5, there is a surprise increase in τ, and this increase is permanent
- In period 5, there is a surprise increase in τ, and this increase will only last until period 10
- In period 0, it is announced that in period 5 there will be a permanent increase in τ
- In period 0, it is announced that in period 5 there will be a 50% chance that τ will increase

6. In an OLG model, the government taxes each young person a fixed amount T. The government invests this tax, paying the person back $(1 + r_{t+1})T$ when they are old. Set up and solve an OLG model incorporating this. Show how this "fully funded" social security system changes the steady state outcome for capital per worker.

7. Individuals live for two periods. They earn labor income of w_t in the first period of their life, and consume in both periods. Their utility function is $U = (1-\beta)\ln c_1 + \beta \ln c_2$, and they take the interest rate of r as exogenous to their consumption decision.

   (a) What is the optimal amount of savings (s_t) done by an individual?

   Production in this economy uses both physical and human capital. The production function is $y_t = k_t^{\alpha} h_t^{1-\alpha}$. The wages of a young person are thus $w_t = (1-\alpha)k_t^{\alpha} h_t^{1-\alpha}$.

   There is a tax on savings at the rate of τ. The proceeds of this tax are used to finance the accumulation of human capital. There is no population growth. Physical capital accumulates as $k_{t+1} = (1-\tau_{t+1})s_t$. Human capital accumulates as $h_{t+1} = x + \tau_{t+1}s_t$. The value x is an amount of exogenously given human capital (basic skills) that is always present.

   (b) Derive an expression for y_{t+1} as a function of y_t.

   (c) What tax rate, τ^*_{t+1}, maximizes y_{t+1}? Draw a graph relating the optimal tax rate to the level of y_t, making sure to indicate the optimal tax as y_t goes to zero and the optimal tax as y_t goes to infinity.

   (d) Assume that in this economy, the tax rate is always optimal. That is, $\tau_{t+1} = \tau^*_{t+1}$ in every period. Does the economy have a steady state growth rate or a steady state level of income?

   (e) Now assume that taxes are set forever at $\tau_{t+1} = 0$ for every period. Does the economy have a steady state growth rate or a steady state level of income?