J Econ Growth
DOI 10.1007/s10887-009-9045-y

The dual economy in long-run development

Dietrich Vollrath

Abstract A salient feature of developing economies is the coexistence of a modern commercial sector alongside a traditional subsistence sector—the dual economy. The apparent differences in productivity between sectors imply substantial losses in aggregate productivity. Existing theories of the dual economy rely on exogenous price distortions, and cannot explain why or if these distortions evolve over the course of development. This paper provides a model of the dual economy in which the productivity differences arise endogenously because of a non-separability between the value of market and non-market time in the traditional sector. Incorporating endogenous fertility, the model then demonstrates how a dual economy will originate, persist, and eventually disappear within a unified growth framework. An implication is that traditional sector productivity growth will exacerbate the inefficiencies of a dual economy and produce slower overall growth than will modern sector productivity improvements.

Keywords Dual economy · Unified growth · Endogenous fertility

JEL Classification O11 · O13 · O17 · O41 · Q10

1 Introduction

Neo-classical growth theory has concerned itself primarily with one-sector models, but most developing countries contain within them multiple economies operating in distinctly different manners and typically with distinctly different levels of productivity. Lewis (1954) brought this concept of dual economies into focus and more recently Banerjee and Duflo (2005) and Temple (2005) have suggested that a better understanding of growth and development requires the explicit adoption of models that incorporate heterogeneity *within* economies.

While the development literature has studied the dual economy closely, what is missing is an analysis of how dualism arises, evolves, and eventually disappears within the overall

D. Vollrath (✉)
University of Houston, 201C McElhinney Hall, Houston, TX 77204, USA

Published online: 10 October 2009

process of development. This paper provides a dynamic model of the dual economy in which the apparently inefficient allocation of labor across sectors is the result of optimizing behavior by individuals. In contrast to previous work, here the extent of duality (as captured by differences in marginal products of workers between sectors) evolves along with the economy, and does not appeal to exogenous institutional differences to explain why some countries display dual economies while others do not. Similar to unified growth models (see Galor 2005 for a complete review) that explain the coincident changes in fertility, education, and income, the model presented here shows how the dual economy is embedded within the process of long-run development.

To achieve this a non-separability is incorporated within a model of optimal time allocation.[1] Individuals will have to choose between market work and a non-market activity. For the traditional sector, individuals internalize the effect of their time allocation on the marginal product of their market labor, realizing that as they work more they are lowering the value of their time. This results in traditional sector individuals allocating less time towards the market activity. Modern sector individuals face a constant wage, the non-separability is not present for them, and they allocate a larger fraction of their time to the market activity. Taking the equilibrium time allocations in the two sectors as given, from the aggregate perspective the marginal product of a traditional *worker* will be lower than the marginal product of a modern sector *worker*, and a dual economy will exist.

The results are not driven by any labor market distortion between sectors, and individuals are freely mobile, ensuring that utility is equalized. The land market is competitive as well, and the result is not driven by the assumption that agricultural workers earn their average, rather than their marginal product. This leads to the implication that the dual economy is not output maximizing, but it is welfare-maximizing. Therefore a dual economy can persist despite the *prima facie* evidence of inefficiency.

Given the basic result, the model can also accommodate an additional feature of the dual economy. As documented below, not only labor productivity levels but fertility rates show variation between sectors. If we assume, as has become standard, that fertility takes time as an input, then one can interpret the non-market activity as being time allocated to child-raising. The model then yields predictions consistent with the evidence: traditional sector fertility is higher than in the modern sector.[2]

An assumption that the non-market activity is related to fertility provides a dynamic structure that can be used to examine how the dual economy evolves in the course of development. Looking at productivity changes (exogenous at first, endogenous in an extension), what is shown is that productivity changes in the two sectors do not have similar effects, a feature shared with the work of Strulik and Weisdorf (2008). Modern sector productivity changes raise the time cost of non-market activity in both sectors, inducing all individuals to spend more time on work and shrinking the gap in worker productivity between sectors. Thus modern sector productivity growth is capable of eliminating the dual economy in the course of

[1] Typically, the separability question in the development literature involves the distinction between labor demand on a family farm versus the labor supply of the family. Benjamin (1992) cannot reject the hypothesis that family labor supply is independent of farm labor demand. However, this result is based on household size being constant. Here, the choice of family size is potentially endogenous and so his findings are not necessarily applicable.

[2] The non-market activity could be interpreted in any of several different ways, however, the dual economy result will follow so long as utility is concave in non-market time. A simple interpretation would be that non-market time is leisure. Alternatively, non-market activity could be associated with home production. Parente et al. (2000) look at the implications of this for cross-country income levels, but do not allow for heterogeneity within the economy. Gollin et al. (2004) also look at home production, additionally assuming that this technology differs between urban and rural areas, but again they do not allow for heterogenous households.

development. This occurs even though the non-separability in the traditional sector is always present.

In contrast, traditional sector productivity growth, due to a low income elasticity of demand for its output, can have a perverse effect on development. An increase in traditional sector productivity will actually lower the time spent working in favor of non-market activity in both sectors, while the gap in productivity between sectors will grow. With the interpretation of non-market activity as fertility, agricultural productivity growth may lower output per capita by increasing population growth. The result is that productivity improvements in the traditional sector can, similar to the work of Matsuyama (1992) and Galor and Mountford (2008), slow down the aggregate growth rate.[3]

The relevance of the dual economy for understanding income differences across countries has been established by several recent empirical papers. Chanda and Dalgaard (2008), Cordoba and Ripoll (2008), Restuccia et al. (2008) and Vollrath (2009) all document that the inefficiencies inherent in a dual economy can explain over half of the observed variation in total factor productivity (TFP) at the country level. Temple and Woessmann (2006) document that labor reallocation has a significant influence on country-level growth rates.[4]

While informative, this literature does not provide any insight on *why* the dual economy exists, nor how it changes over time. Development theory generally focuses on the static problem of how wage differentials could arise and persist, as in the Harris and Todaro (1970) model of migration and Stiglitz (1974) efficiency wage theory, but these models do not explore how duality might change over time. More recently, dynamic treatments of two-sector economies provided by Hayashi and Prescott (2008), Caselli and Coleman (2001), and Mourmouras and Rangazas (2007) have included dual economy effects, but they all rely on exogenously given distortions to prices or preferences to generate their results, similar to the earlier efforts of Jorgenson (1961) and Ranis and Fei (1961).[5] The current work differs in that the results do not depend on an exogenously fixed friction.[6]

From a historical perspective, the model is consistent with recent work by Clark (1999) that downplays the role of agricultural productivity changes in generating the Industrial Revolution. Clark finds little evidence for massive productivity changes in agriculture in the period 1600–1914. The expansion of agricultural output occurring alongside the rapid urbanization of the British population in the 19th century could instead be located in an “Industrious Revolution”, as in de Vries (1994), who suggests that the crucial change in industrializing Europe was the switch to less leisure (here, less non-market activity) and more market production. What the current model suggests is the the Industrious Revolution was a consequence of increasing manufacturing productivity itself, rather than a cause.

[3] The implications of this model are also related to several papers on endogenous fertility and income distribution, as in Dahan and Tsiddon (1998) and de la Croix and Doepke (2003). Here, duality creates the possibility of higher fertility and lower income levels, while in these other models differential fertility due to human capital differences may limit development.

[4] This recent work is an outgrowth of an older literature on structural transformation (Chenery and Syrquin 1975; Chenery et al. 1986; Kuznets 1966) focused on the movement of labor from agriculture to industry as an important source of growth in output per capita.

[5] Proto (2007) explores situations in which the initial wealth distribution leads to duality in skill acquisition.

[6] The literature on economic growth has numerous examples of multi-sector models. Matsuyama (1992), Laitner (2000), Gollin et al. (2002) and Kongasmut et al. (2001) examine labor movements between sectors, but no duality is allowed to exist. Unified growth models such as Galor and Mountford (2008), Galor et al. (2009), Goodfriend and McDermott (1995), Tamura (2002), and Hansen and Prescott (2002) involve the transition between sectors, but do so without exploring the dual nature of the economy. Kogel and Prskawetz (2001) present a growth model in which agricultural workers earn their average product while industrial workers earn their marginal product, but the ramifications of this are not explored.

Aside from the implications regarding productivity, an important item to note is what the model tells us about evaluating dual economies. We can establish that the dual economy has consequences for aggregate output per capita, but we cannot infer welfare implications from this.

To proceed, Sect. 2 discusses the evidence regarding the dual economy and its manifestation in productivity and fertility differences between sectors of economies. Section 3 then presents the general model of duality, showing how it arises endogenously from a model of optimal time allocation. The dynamic implications of the dual economy are examined in Sect. 4 by explicitly making non-market activity equal to fertility, including an extension of the model to allow for endogenous productivity growth in a unified growth framework. Section 5 concludes.

2 Dualism in production and fertility

It will be useful to pursue a digression on semantics before reviewing the evidence on dualism. The phrase "dual economy" has several connotations. 'Modern sector dualism' typically deals with the under-employment of labor in the urban sector, while 'traditional sector dualism' focuses more on the coincidence of a low productivity traditional sector alongside a modern, commercially oriented sector.[7] In this paper, I focus exclusively on the latter type of dualism, laying aside urban labor market issues completely.

What is meant by the 'traditional sector'? Numerous definitions are available, but typically this refers to a sector of production that is predominantly rural, is limited by some fixed resource (e.g. land), is engaged in subsistence rather than commercial production, and is demonstrably poorer than the 'modern' sector (see Lewis 1954; Ranis 1988; Basu 1997).

In the work that follows I will use the term 'agricultural' to refer to a sector that is generally rural and that relies heavily on certain fixed factors of production such as land. I will use the terms 'modern' or 'manufacturing' to refer to the sector that is generally urban and commercially oriented. These labels are for convenience only, and are not meant to imply any strict divisions.

2.1 Differential productivity

The most obvious feature of the dual economy is the pronounced difference in productivity between the agricultural and modern sectors. The (presumed) relatively low marginal product of agricultural laborers suggests that some improvement to living standards could be achieved by the transfer of labor into the modern sector.

The *prima facie* evidence of these productivity differences comes from the observation that output per worker in agriculture is typically much lower than that of the manufacturing sector. Figure 1 plots the log of the relative labor productivity between the manufacturing and agricultural sectors against the log of income per capita for a cross-section of countries. As can be seen, this ratio declines markedly as income increases.

Similar evidence is presented in Temple (2005, see Appendix B), who documents that the relative labor productivity in manufacturing is nearly six to one for Sub-Saharan African countries in 1996, and even 1.7 to one in the OECD. As he points out, though, differences in average products do not necessarily mean that marginal products differ.

[7] See Bertrand and Squire (1980) for a more careful definition of these terms.

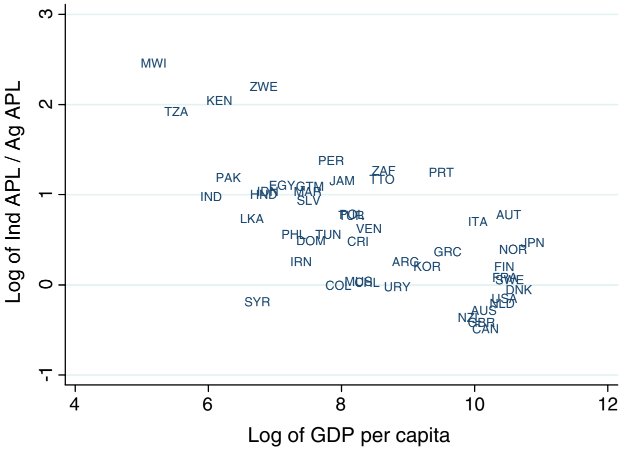

Fig. 1 Relative labor productivity and income per capita. *Note*: data are from Vollrath (2009), and are derived from Penn world table information on PPP levels of GDP and world bank data on domestically priced output by sector that is converted to PPP using an adjustment described fully by Caselli (2005)

Assume that both agriculture and manufacturing have Cobb-Douglas production functions, and let labor's share in agriculture equal θ_A, while it's share in manufacturing is θ_M. The ratio of marginal products is therefore

$$\frac{MPL_M}{MPL_A} = \frac{\theta_M}{\theta_A}\frac{APL_M}{APL_A}. \tag{1}$$

If it is the case that $\theta_M < \theta_A$, then it is theoretically possible for $APL_M > APL_A$ to be consistent with equality in marginal products. Duality, as traditionally understood, would not hold because there was no scope for increasing output by transferring labor between sectors.

However, it is hard to actually reconcile the observed gaps in average products with the equalization of marginal products. If we assume, as we do for aggregate production functions, that the labor share in output is essentially equal across countries, then this suggests that θ_M/θ_A is constant across countries. If this is true, then it is impossible for *all* countries to have $MPL_M = MPL_A$ given that their ratios of average product differ.

Perhaps this just indicates that it is not true that θ_M/θ_A is the same in every country. If we still want to assert that marginal products are equalized across sectors, then it must be that θ_M/θ_A is the reciprocal of the ratio of average products. For several developing nations, the ratio of average products is on the order of 8 to one. Therefore θ_M must be only one-eighth of θ_A. If labor's share in agriculture is 0.6 (see Jorgenson and Gollop 1992), then this implies that labor's share in manufacturing is only 0.075. This seems absurdly low. If we want to assert that labor's share in manufacturing is equal to only 0.2, we have the nonsensical result that the labor share in agriculture must be equal to 1.6, or that labor earns over 100% of total output.

So while the observations on average product cannot directly show that marginal products differ between sectors, it is hard to escape the conclusion that they do. Previous models of the dual economy have used exogenous institutional distortions to explain why wages may persistently differ between sectors, but they are not capable of explaining the relationship between income and duality in Fig. 1.

One possibility is that the dual economy and overall poverty are both outcomes of some deeper structural flaw or institutional failure across countries. However, the inverse relationship of relative productivity and income holds over time even for those countries that are the richest today.

David (2005), in his re-examination of the data on real income in the early years of the United States, finds evidence of a dual economy. In 1840, the relative labor productivity of agricultural workers (narrowly defined) was only 40% of non-agricultural workers. Even on a per man-hour basis, the ratio is still 77%. Over the whole period of 1790–1860, David concludes that the average product of labor was relatively low in the agricultural sector, but that this was primarily accounted for through fewer hours of work rather than lower productivity per hour. This difference between the agricultural and modern sectors will arise endogenously within the model presented here, and just as David documents for the U.S., a portion of the increase in output per capita will result from an increase in hours worked by all individuals as well as a shift of workers from agriculture to manufacturing.

2.2 Differential fertility

With an appropriate interpretation of non-market time, the model presented will be able to account for the observed fertility differences between sectors that occur alongside the gaps in labor productivity.

For the United States, Grabill et al. (1958) report fertility data which covers the whole period of industrialization in the United States, 1800–1950.[8] The overall decline in fertility in this period can be broken into three components: a decline in urban fertility, a decline in rural fertility, and the shift of population from rural to urban areas. Grabill et al. (1958) calculate the contribution of each factor to the overall decline in this period. They find that about 56% of the decline is attributable to the fall in rural fertility, with 24% accounted for by urban fertility decline and 20% to the shift from rural to urban areas.[9] So while there is a differential in fertility between sectors, over time this differential shrinks and fertility converges to a uniformly low level.

This pattern is repeated in the European experience, as summarized by Sharlin (1986) using the data from the Princeton European Fertility Project. Sharlin confirms that rural marital fertility was indeed higher than urban marital fertility at virtually every date that data were collected. When the demographic transition began, it began in the urban areas before spreading to the rural sector. This trend is confirmed in the more detailed studies of Germany by Knodel (1974) and Belgium by Lesthaeghe (1977).

As the demographic transition continues, fertility remains always higher in rural areas but the differential decreases. The pattern is thus similar to what we identified above for the United States—a convergence of rural fertility to urban fertility. Sharlin also finds that the declines in both rural and urban fertility play the predominant role in lowering overall fertility, confirming the finding in the U.S. that the shift of population from rural to urban areas was of secondary importance.

One interesting aspect of fertility behavior that is highlighted by Sharlin is the effect of city size. Knodel (1974), Lesthaeghe (1977) and Livi-Bacci (1977) all present evidence that as the population of the urban area increases, fertility rates fall. This effect appears to be tied

[8] Due to data limitations in the U.S. Census their fertility measures are child/woman ratios. This ratio is not ideal because it does not control for age structure, but it is available and does provide a decent portrayal of the experience of American fertility over time.

[9] These percentages are tied to the time period in question. Using a later beginning date would lower the initial rural population share and decrease rural fertility's role in the overall decline, while raising urban fertility's role. Grabill et al. (1958) do not find changing the time frame alters significantly the share of overall decline caused by rural to urban shifts.

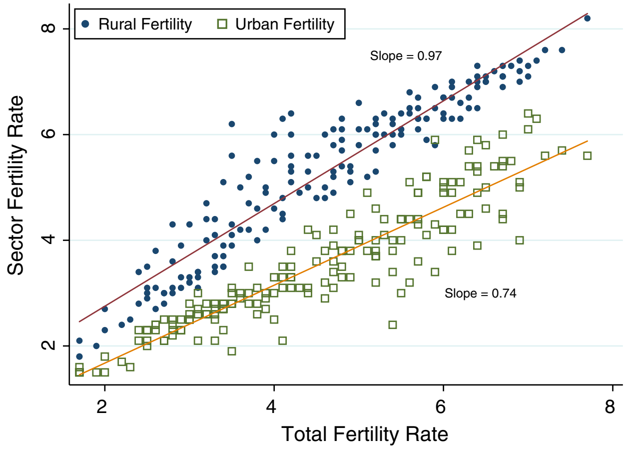

Fig. 2 Sector fertility rates and total fertility. *Note*: data is from the demographic and health survey (DHS), showing urban and rural fertility rates for each of 172 different surveys from 74 different developing countries in various years. The slopes are estimated by OLS

to the nature of work in these areas, as those places engaged in more industrial work—urban or not—are more likely to have lower fertility. Thus Sharlin suggests that if we could look at the fertility of agricultural and non-agricultural workers we would find higher differentials than those found between urban and rural areas. The model presented in this paper fits neatly into this framework, as the distinction is truly between agricultural and non-manufacturing workers, and actual residence is not important.

These long-run experiences of the currently developed world show that duality in fertility can exist and then disappear over time, but do currently developing countries fit within this framework? One of the earliest looks at the rural-urban differential comes from Kuznets (1974). He considers a cross-section of nations from the late 1950's and early 1960's. Using child/woman ratios he calculates the rural-urban differential and finds that rural areas are consistently more fertile, but that the gaps are not terribly large.[10]

Similar findings come from another cross-sectional study by Findley and Orr (1978) which looks at fertility behavior around 1970 in thirty-eight developing countries. They find again that differentials in rural-urban fertility rates are closely associated with a countries overall position in the demographic transition. In their seven highest fertility countries, there is essentially no differential. As they examine countries with lower overall fertility the differential widens substantially. For the lowest fertility countries they examine—with overall total fertility rates (TFR) of around 4.5-urban TFR is only 60% of rural TFR.

More recent data confirms the dual nature of fertility, and the convergence of fertility rates as economies develop. Figure 2 plots urban and rural fertility rates for all of the surveys undertaken by the Demographic and Health Survey project. As can be seen, rural fertility is consistently higher than urban fertility in this set of developing countries. Additionally, as the total fertility rate falls, rural fertility falls faster than urban fertility, indicated by the higher slope for the rural observations.

[10] Kuznets does find several countries in Africa in which the urban child/woman ratio is actually higher in urban areas than in rural ones, but he points out this could be caused by differences in age structure which the child/woman ratio does not account for.

3 The dual economy

The evidence reviewed provides several distinct stylized facts that must be accounted for when trying to model a dual economy. First, differences in labor productivity (and likely in marginal productivity) exist between the agricultural and the modern sector. Secondly, these differences in productivity evolve along with the economy, shrinking in size at higher levels of aggregate output per capita.

Similarly, fertility differences exist within the economy, and as the economy develops both rural and urban fertility decline. Rural fertility appears to fall faster, though, so that the sectoral rates converge.

To explain these facts, the model presented here is based upon an individual's optimal use of their time endowment. One option is to work producing a marketable output that can be exchanged for or used directly for consumption. The second option for the use of time is in a non-marketable activity that generates utility for the individual. One could interpret this non-market activity in various ways: leisure, home production, and fertility would be common examples. As long as the utility of this activity is concave in non-market time, the dual economy result will hold.

To actually generate differences between sectors, I introduce a non-separability into the time allocation decision for individuals in the traditional, agricultural, sector.[11] Non-separability implies that individuals in the sector appreciate the effect of their time allocation choice on the marginal product of their time. They will see a rising marginal product of labor as they spend more time producing agricultural goods, and in equilibrium this will lead to traditional individuals spending less time on market work.

The non-separability implies that the agricultural sector has no labor market, a useful abstraction consistent with the findings of the development literature. The standard argument is that transaction costs, in particular those related to supervision costs for hired labor, dominate any potential economies of scale in agricultural production (Binswanger and Rosenzweig 1986; Roumasset 1995). Given these supervision issues, agriculture is dominated by "family farms" operated as autarkic production units. Eastwood et al. (2008) show that this type of organization is the dominant form across nearly all countries, excepting the most highly developed nations. Labor hiring is not completely absent (Rosenzweig 1988), but the informational issues present in supervision generally lead to land-owners making linked transactions that yield managerial oversight to tenants in order to maximize production. Eswaran and Kotwal (1985) argue that it is the lack of markets in both labor effort and managerial skill that lead to family farms dominating production. While assuming that *no* labor market exists is simplistic, it does appear to capture the general nature of the agricultural sector in developing areas.

To begin the model, consider a two-sector economy consisting of L people. Of these people, a share $a \in (0, 1)$ work in the agricultural sector while the remaining $1 - a$ work in manufacturing. As noted in Sect. 2, the labels agriculture and manufacturing are for convenience, and could alternately be labeled the "traditional" and "modern" sectors. Ultimately, the important distinction is between individuals facing a non-separability due to a missing labor market and those who work in a sector with such a market.

Each individual has a unit of time, with the share $s \in (0, 1)$ allocated to productive work, and the remaining $1 - s$ time spent in non-market activity. Individuals must consume a fixed amount of agricultural goods, $\overline{b}$, at a relative price p^A. They receive utility from the amount of manufacturing goods they consume as well as from their non-market time. Their decision

[11] A simpler form of this type of model can be found in Weisdorf (2006), who does not consider explicitly the dual nature of the economy or the dynamic implications.

problem is thus one of allocating time between market and non-market work, conditional on working enough to acquire the subsistence amount of agricultural goods.

An important assumption is that individuals only work in one sector, and cannot split their time between agriculture and manufacturing. This assumption is a necessary, but not sufficient, condition for generating a dual economy result. It should also be noted that this assumption is not unique to this paper, and any model attempting to provide an explanation of duality would need something similar.[12]

3.1 Production

Agricultural production is a constant returns to scale function of labor effort and land. Capital is ignored throughout to simplify the analysis, but this does not materially alter the results. Total agricultural production is denoted as

$$Y_t^A = A_t^A F(R, E_t^A) \tag{2}$$

where A_t^A is total factor productivity in the agricultural sector, R is the total amount of land (resources) in the sector, and $E_t^A = s_t a_t L_t$ is the total labor effort expended. F is constant returns to scale, and has typical concave properties

$$F_R > 0, F_{RR} < 0, F_E > 0, F_{EE} < 0, F_{RE} > 0. \tag{3}$$

Each of the $a_t L_t$ individuals working in agriculture operates as an independent firm, renting in land for use. Net income for a representative farmer is

$$I_t^A = p_t^A A_t^A F\left(r_t, s_t\right) - \rho_t r_t \tag{4}$$

where r_t is the land employed by the farmer, and ρ_t is the rental price of land, which is taken by the farmer as given. The price, p_t^A, is the price of agricultural goods relative to manufacturing goods.

Farmers will maximize their net income by setting the marginal product of land equal to ρ_t. In equilibrium it must be that all land is employed, and given that every farmer is identical the allocation of land to each farmer is $r_t = R/a_t L_t$. The equilibrium rental price of land is therefore

$$\rho_t = p_t^A A_t^A F_R\left(\frac{R}{a_t L_t}, s_t\right). \tag{5}$$

With constant returns to scale in the production function, this means that the net income of an individual farmer can be written as

$$I_t^A = p_t^A A_t^A F\left(\frac{R}{a_t L_t}, s_t\right) - \rho_t \frac{R}{a_t L_t} = s_t p_t^A A_t^A F_E\left(\frac{R}{a_t L_t}, s_t\right) \tag{6}$$

which says simply that the net income of a farmer is equal to the marginal product of their time multiplied by the total time they spend working.

A typical method of introducing a dual economy effect is to presume that agricultural workers earn the average product of their labor, rather than the marginal. This often goes hand in hand with the assumption of no property rights over land. That is not necessary in

[12] As a justification, consider imposing a time cost on individuals who work in both sectors. This could arise because of transportation costs or because their inability to specialize makes them less productive. Regardless, for a sufficiently sized time cost, it would be optimal for individuals to work in only one sector.

this model to achieve the dual result, though. Here, the land market exists, farmers pay rent, and ultimately earn their marginal product.

What is central to the results is the presumption that farmers each work individually. They will internalize their choice of s_t on their net income, realizing that working more will lower the marginal product of their time. This will lower the time cost of non-market activities, and therefore agricultural workers will optimally choose to work less. If agricultural "firms" organized production, paying rents for land and wages for work, then the agricultural workers would act identically to industrial sector workers, because they would not internalize their choice of s_t on the wage.

The manufacturing sector is presumed to be perfectly competitive, so that labor effort is paid its marginal product. For the main results, it is not necessary to completely specify the manufacturing production function. It will suffice to say the wage rate per unit of effort is

$$w_t^M = A_t^M w(a_t) \tag{7}$$

so that the wage rate depends on the productivity of the manufacturing sector, A_t^M, and also on a function of the number of people in agriculture, $w(a_t)$. The function $w(\cdot)$ is presumed to have the following properties: $w' > 0$ and $w'' < 0$.

These properties imply that the manufacturing wage increases as the number of people in manufacturing $(1 - a_t)$, decreases. Net income for a manufacturing worker is simply

$$I_t^M = w_t^M s_t \tag{8}$$

and the central assumption made here is that manufacturing workers do not internalize the effect of their work effort on the wage rate w_t^M.

3.2 Individual optimization

Individuals at time t receive utility from their consumption of the manufactured good (c_t) and from the amount of non-market time they have $(1 - s_t)$. Utility is

$$U_t = c_t + V(1 - s_t) \tag{9}$$

with the properties that $V' > 0$ and $V'' < 0$, or there is diminishing marginal utility to non-market activity. If one were to assume that non-market activity was equivalent to fertility, this would be similar in form to the work of Strulik and Weisdorf (2008), This quasi-linear utility is a convenient way of capturing a "hierarchy of needs" such that non-market activity is a higher priority than consumption of manufacturing goods, and there are no direct income effects on the time allocation. If non-market activity is home production, then this form of utility suggests that there are always some home activities undertaken before an individual will enter market production.

Income for an individual includes the labor income earned, either I_t^A or I_t^M depending on the sector they work in, plus rents earned on ownership of land. For simplicity, land is presumed to be owned equally across all individuals alive in period t. Each individual earns $\rho_t R/L_t$ in rents.[13]

In terms of consumption, each individual must fulfil a basic subsistence constraint first, eating $\overline{b}$ units of the agricultural output. Once they have met this requirement, their consumption is given by

[13] One could allow for inequality in land-ownership. In the subsequent work, this would only require that utility is equalized across sectors for individuals with identical land-holdings. All the results would remain valid.

$$c_t = I_t^k - p_t^A \bar{b} + \rho \frac{R}{L_t} \tag{10}$$

where $k \in (A, M)$ refers to the sector in which an individual works.

Utility maximization over the time allocation, s_t^A for an individual in the agricultural sector yields a first order condition of

$$p_t^A A_t^A (F_E + s_t^A F_{EE}) = V' \tag{11}$$

while the first order condition for manufacturing workers is

$$w_t^M = V' \tag{12}$$

and the essential difference between the two sectors can be seen. For manufacturing workers, the marginal cost of additional non-market time is simply equal to the wage. Agricultural workers, though, internalize the effect of their decision on the marginal value of their time. The marginal cost of non-market time is lower because individuals realize that by working less they are increasing their marginal product. This follows from the fact that $F_{EE} < 0$.

What these first order conditions show is that even if the marginal product of a unit of time is identical between sectors, $p_t^A A_t^A F_E = w_t^M$, agricultural workers will make different choices from manufacturing workers regarding time allocations.

3.3 Equilibrium

Recall that all individuals must consume a quantity $\bar{b}$ of the agricultural good. Therefore, it must be the case that

$$\bar{b} L_t = A_t^A F\left(R, s_t^A a_t L_t\right) \tag{13}$$

which states that total agricultural production must be sufficient to provide the subsistence consumption to all individuals. Given the choice of time allocation in agriculture the share of individuals employed in agriculture, a_t, is fixed by this equation.

Individuals are assumed to be freely mobile between sectors, and this will ensure that the utility of individuals in the two sectors is equalized, or $U_t^A = U_t^M$. As noted previously, it is assumed that individuals only work in a single sector at a time. With this, a definition of the equilibrium can be provided.

Definition 1 At time t, define an **equilibrium** as a set of prices and allocations $\left(p_t^{*A}, a_t^*, s_t^{*A}, s_t^{*M}\right)$ that, given the state of the economy $\left(A_t^A, A_t^M, R, L_t\right)$, fulfills the following four conditions

- s_t^{*A} maximizes U_t^A
- s_t^{*M} maximizes U_t^M
- $U_t^A = U_t^M$
- $\bar{b} L_t = A_t^A F\left(R, s_t^{*A} a_t^* L_t\right)$

At this point it is possible to demonstrate that a dual economy situation exists, which is spelled out in the following proposition.

Proposition 1 *In equilibrium, a dual economy exists. Specifically this means that the following hold:*

(A) Manufacturing workers allocate more time to productive work than agricultural workers, $s_t^{*M} > s_t^{*A}$

(B) *The marginal product of a* ***worker*** *is higher in the manufacturing sector,* $w_t^M s_t^{*M} > s_t^{*A} p_t^{*A} A_t^A F_E\left(\frac{R}{a_t^* L_t}, s_t^{*A}\right)$

Proof See Appendix □

The proposition makes clear the effect of the differences in the first order conditions shown in (11) and (12). Because they understand the implications of their actions, agricultural workers optimally choose to work less and enjoy more non-market time. This difference means that on a per worker basis, agricultural workers are marginally less productive. That is, the additional output of adding another agricultural worker, given s_t^{*A}, is less than the additional output gained from adding another manufacturing worker. This discrepancy in marginal products is the subject of the following corollary.

Corollary 1 *Given the results of Proposition 1, output* ***per worker*** *can be increased by a transfer of labor from agriculture to manufacturing.*

Proof This follows from part (B) of Proposition 1, which states that the marginal product of agricultural workers is lower than workers in manufacturing. Therefore, transferring labor from agriculture to manufacturing will increase output per capita. □

It is crucial to note that while Corollary 1 says that a transfer of labor can increase *output*, it does not increase *welfare*. This explains why a dual economy situation may persist despite the *prima facie* evidence that the economy is inefficient. Corollary 1 thus provides a potential explanation for the cross-country results highlighted in the introduction.

In addition, the results of this model provide an explanation for why the dual economy will persist over time despite the apparent benefits to agricultural workers of moving to the manufacturing sector. Prior explanations for the persistence focused on explaining what kind of frictions had to exist to keep agricultural workers from moving (e.g. unemployment in urban areas, migration costs, etc.). In contrast, the current model can explain the persistence of the dual economy without having to introduce any such exogenous costs.

3.4 Productivity changes

Another advantage of this model is that it offers a natural explanation for how the dual economy evolves with changes in the economy at large. Here, the degree of difference between the sectors is directly related to productivity in the different sectors. Thus the model can provide an explanation not only for why the dual economy exists, but why it tends to disappear as economies develop. In doing so, it will show that productivity changes in agriculture and manufacturing are different in their effects.

Proposition 2 *Given agricultural production in (2) and manufacturing wages in (7),*

(*A*) *An increase in* A_t^A *has the following effects:*

- *The share of labor in agriculture,* a_t^**, falls*
- *The time allocations to market work,* s_t^{*A} *and* s_t^{*M}*, both fall*

(*B*) *An increase in* A_t^M *has the following effects:*

- *The share of labor in agriculture,* a_t^**, falls*
- *The time allocations to market work,* s_t^{*A} *and* s_t^{*M}*, both rise*

Proof See Appendix □

From Proposition 2, it becomes clear that the kind of technological progress experienced by an economy matters for long-run output per capita. If agricultural productivity increases, labor is "pushed" into the manufacturing sector, the typical result in two-sector models. However, by lowering wages in the manufacturing sector this reduces the marginal utility of work and raises non-market activity in the manufacturing sector. The agricultural sector responds by increasing non-market activity as well, as the marginal productivity of their work must fall to ensure that labor is willing to flow into the manufacturing sector. The decreases in sector-level market work act to dampen the effect of the shift into manufacturing so that at the aggregate level it is unclear whether total market work time will go up or down.

In contrast, an increase in manufacturing productivity induces an increase in work effort in that sector. In response to the higher utility in manufacturing, labor is "pulled" out of agriculture. To maintain agricultural output, time spent in market work by agricultural individuals has to increase. Overall, the aggregate time spent on market work increases. It is this effect that calls to mind the "Industrious Revolution" of de Vries, but note that here the increase in market work effort is a consequence, not a cause, of increased productivity.

An additional result can be established regarding the dual economy.

Corollary 2 *Given the results of Proposition 2,*

(*A*) *An increase in agricultural productivity, A_t^A:*

- *Increases $s_t^{*M} - s_t^{*A}$, the difference in work effort between sectors*
- *Increases $w_t^M s_t^{*M} - s_t^{*A} p_t^{*A} A_t^A F_E\left(\frac{R}{a_t^* L_t}, s_t^{*A}\right)$, the difference in the marginal product of a worker between sectors.*

(*B*) *An increase in manufacturing productivity, A_t^M:*

- *Decreases $s_t^{*M} - s_t^{*A}$, the difference in work effort between sectors*
- *Decreases $w_t^M s_t^{*M} - s_t^{*A} p_t^{*A} A_t^A F_E\left(\frac{R}{a_t^* L_t}, s_t^{*A}\right)$, the difference in the marginal product of a worker between sectors.*

Proof See Appendix. □

What the corollary shows is that the extent of the dual economy changes based on the relative productivity of the two sectors. An agricultural productivity increase, while decreasing the size of the agricultural sector, exacerbates the differences between the sectors. Sustained increases in agricultural productivity will industrialize the economy, but this will be accompanied by a growing disparity in measured productivity between sectors.

In contrast, manufacturing productivity increases will not only industrialize the economy, but induce agricultural workers to act more like manufacturing workers. Industrialization thus accompanies the disappearance of the dual economy in this case.

What Proposition 2 and Corollary 2 establish is that the long-run impact of productivity growth depends on the sector in which it arises. In typical models, increasing agricultural productivity causes industrialization. What these results indicate is that some of this industrialization will be offset by decreases in market work effort. At the aggregate level, measured output per capita may not increase following agricultural productivity changes. If non-market activity is associated with home production, this could be simply because more effort is, by definition, becoming unmeasured. If non-market activity is associated with fertility or leisure, then this may actually reflect a decrease in output per capita. In any case, though, we cannot presume that a decrease in measured output per capita implies a decrease in welfare.

4 Fertility and the dynamics of development

The economy described in the previous section shows that dualism can result even with optimizing agents and fully functioning land markets once we allow for the non-separability in the agricultural sector.

A study of the dynamics of duality will depend on exactly how one interprets the non-market activity. Parente et al. (2000) and Gollin et al. (2004) focus on this as home production. Both papers look at how this production relates to distortions in capital accumulation, but leave heterogeneity and population dynamics aside.

Similar to work on unified growth (see Galor 2005) I will focus here on the role of fertility. This approach is taken so that the dual economy can be examined over the long run in which changes in population growth rates are relevant. A very common assumption in the unified growth literature is that the input to fertility is time. Therefore one can easily interpret the non-market activity modeled above as an optimal fertility choice, where fertility is increasing in non-market time. The quasi-linear utility is similar to that of Strulik and Weisdorf (2008) in their model of fertility and growth.

The first thing to note is that such an interpretation means the dual economy model is perfectly consistent with the evidence on fertility by sectors presented in Sect. 2. There, recall, rural/agricultural fertility was always higher than urban/manufacturing fertility. Proposition 1, part (A), shows that manufacturing workers allocate more time to market work, and therefore must spend less time raising children than agricultural workers. The duality in fertility can be explained as an additional result alongside the duality in productivity.

Now, the implications of Proposition 2 and Corollary 2 are that the *type* of productivity growth can have important consequences for fertility. If agricultural productivity goes up, fertility in each sector will actually *increase*. As individuals switch from the high-fertility agricultural sector to the low-fertility manufacturing sector, the effect on aggregate fertility is indeterminate. In contrast, an increase in manufacturing productivity will lower fertility in both sectors and due to the shift in labor to manufacturing aggregate fertility will fall as well. Importantly, note that as manufacturing productivity increases the gap in fertility between sectors will narrow, allowing for the convergence of fertility rates during development.

To see how the introduction of endogenous fertility to the dual economy manifests itself over time, in this section I provide several simulations that incorporate different patterns of productivity change (both exogenous and endogenous) to the model. Fertility is presumed to be linearly increasing in non-market time, so that an individual in sector i has $n_t^i = \overline{n}(1-s_t^i)$. The value $1/\overline{n}$ captures the time cost of children.

The specific production and utility functions, as well as the algebra solving the model have been relegated to the Appendix. The most important point is that manufacturing output, for simplicity, is presumed to be linear in labor effort (and capital is excluded from the analysis). This makes the problem tractable, but removes the possibility that agricultural productivity growth will lower the optimal time allocation in manufacturing by lowering the marginal product in that sector. If this effect were present, then the potentially detrimental effects of agricultural productivity growth explored in this section would only be strengthened.

Several types of simulations are considered. In the first, two initially identical economies experience exogenous productivity "revolutions", one in the agricultural sector and

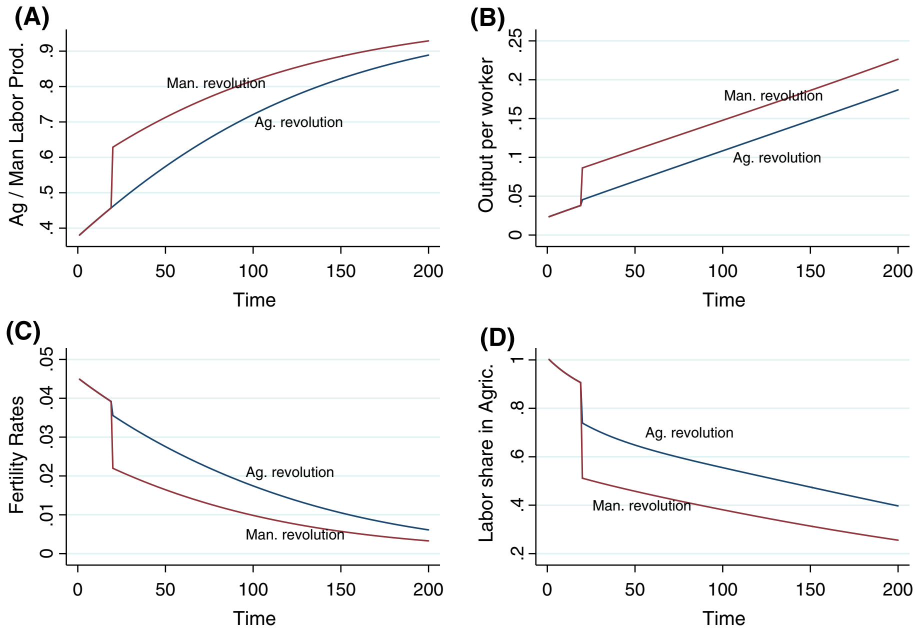

Fig. 3 Productivity revolutions. *Note*: the panels track the evolution of various characteristics under two different regimes of productivity revolutions. "Ag. revolution" refers to an agricultural total factor productivity growth (TFP) of 15% in period 20. "Man. revolution" refers to manufacturing TFP growth of 15% in period 20. Productivity growth in both sectors in both economies is equal to 0.2% in all other periods

one in the manufacturing sector. For the second, sustained differences in exogenously given productivity growth rates are examined for their affect on development.

Finally, productivity growth in both sectors is made endogenous, and the simulation shows how the dual economy evolves over time from the initial Malthusian era to the period of sustained growth. This simulation highlights how the dual economy can be seen as simply another feature of the process of long-run development, alongside fertility change and the structural transformation.

4.1 Productivity revolutions

In this simulation, two economies are compared. Both have some latent exogenous productivity growth of 0.2% per year in each sector. The economies vary only in a productivity shock that strikes them at period 20 (a total of 200 periods are simulated):

- *Agricultural Revolution:* a 15% increase in A_t^A
- *Industrial Revolution:* a 15% increase in A_t^M

At the time of the shock, both simulations have an agricultural share of labor of approximately 70%, or they are relatively under-developed.

Figure 3 plots the results of the simulations for several important variables. Panel (A) shows the relative labor productivity of agriculture to manufacturing. As can be seen, both sectors begin with a distinct dual economy, with agricultural labor productivity only about 40% of that in manufacturing. With an Industrial Revolution, though, this ratio jumps distinctly to nearly 65% and is always higher than when an Agricultural Revolution occurs.

The Agricultural Revolution has no effect on the dual economy at first because agricultural productivity changes do not induce higher work efforts.

Panel (B) shows how the two revolutions influence output per capita. As can be seen, the Industrial Revolution puts the economy on a permanently higher level of output per capita, while the growth rates remain identical. The source of this advantage is the increase in work effort that the Industrial Revolution initiates relative to the Agricultural. The higher work efforts also lead to lower fertility, as seen in figure (C). This lower fertility allows the economy with the Industrial Revolution to retain its higher output per capita by reducing the size of the population relative to the resource endowment.

Finally, panel (D) shows a somewhat counter-intuitive result. The Agricultural Revolution reduces the share of labor employed in that sector; this is the typical "push" idea that normally informs multi-sector models of industrialization. However, the Industrial Revolution produces an even larger drop in the agricultural labor share, even though they do not experience any significant increase in productivity in that sector. The difference comes from the increased work effort induced by the Industrial Revolution.

When A_t^M jumps by 15%, this increases the marginal value of time for manufacturing workers, and because of the increase in the relative price of agricultural goods, for agricultural workers as well. Thus the optimal allocation of time shifts towards work and away from fertility in both sectors. Given the concave nature of utility from fertility, agricultural workers have to increase their time allocation to work by more than manufacturing workers to achieve the same increase in marginal utility. This narrows the gap in their work effort, and narrows the dual economy effect. In addition, the increased work effort in agriculture acts like a productivity increase, lowering the share of people necessary to provide agricultural goods. Thus an Industrial Revolution can induce a greater structural transformation in the dual economy than a similar Agricultural Revolution is capable of.

4.2 Differential growth rates

Rather than examining singular "revolutions" in productivity, in this section the development of economies is tracked under sustained differences in productivity growth across sectors. The initial conditions of the simulations are identical to those of the previous section. The only difference is that productivity growth is concentrated in one sector versus the other in the two parallel simulations.[14]

- *Agricultural-led growth:* A_t^A grows at the rate of 1.5% per period, while manufacturing productivity A_t^M grows only at 0.1% per period.
- *Manufacturing-led growth:* A_t^A grows only at 0.1% per period, while A_t^M grows at 1.5% per period.

From this common starting point, each simulation is run forward for 200 periods. Figure 4 compares the path of several variables across the different productivity regimes. Panel (A) shows the ratio of agriculture output per worker to manufacturing output per worker, the *prima facie* evidence of a dual economy. As can be seen, in period zero under both regimes this ratio is only 0.38. Very quickly, though, manufacturing-led growth increases this to nearly one. In contrast, agricultural-led growth, by limiting the changes made to time allocations, shows only a very slow increase in the relative productivity of agricultural workers. Thus the dual economy persists much longer under agricultural growth.

[14] The exact growth rates used in these simulations are not crucial, and were chosen solely to highlight the distinction in performance.

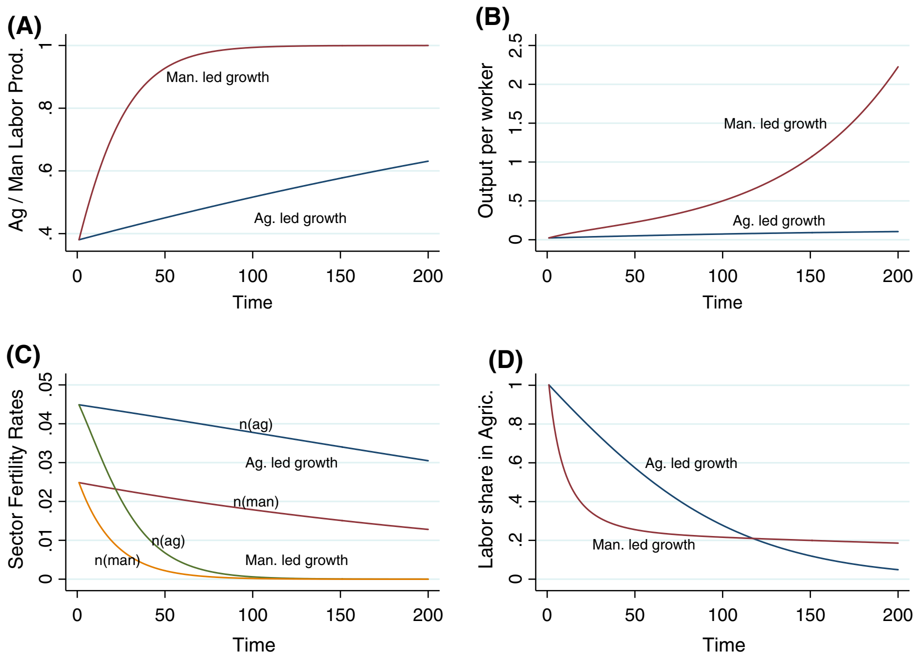

Fig. 4 Development under different productivity regimes. *Note*: the panels track the evolution of various characteristics under two different regimes of productivity growth. "Ag. led growth" refers to agricultural total factor productivity growth (TFP) of 1.5% per year, while manufacturing TFP grows at 0.1% per year. "Man. led growth" refers to agricultural TFP growth of 0.1% and manufacturing TFP growth of 1.5% per year

The consequences of this persistence are apparent in panel (B). Here output per worker is tracked, and under both regimes it begins at a value of 0.023. By the end of the simulation output per capita is nearly 5 times larger under agricultural growth, but this is overwhelmed by the nearly 100 fold increase in output per capita under manufacturing-led growth.

One of the main reasons for this disparity is the relative fertility levels in the two regimes. Panel (C) shows how both agricultural and manufacturing fertility change over time. With agricultural-led growth, fertility in both sectors declines slowly over time, leading to large population increases that literally eat up much of the productivity benefits of agricultural productivity change. In contrast, manufacturing-led growth shows rapid decreases in fertility in both sectors as people allocate more time to work when A_t^M goes up quickly. As a result, the manufacturing-led growth regime is trying to support fewer individuals, and this offsets their lower agricultural productivity level. Note as well that manufacturing-led growth shows a convergence of fertility rates between sectors while agricultural-led growth does not demonstrate this.

Finally, panel (D) plots the share of labor engaged in agriculture, which begins at essentially 100%. With manufacturing-led growth, this share drops very quickly, showing earlier industrialization than the agricultural-led regime. However, the manufacturing-led regime only slowly declines below 20% in the long run as the relatively low level of A_t^A in period 200 means a larger fraction of individuals must remain in that sector. In the agricultural-led regime, industrialization occurs more slowly, but ultimately is nearly complete.

Overall, the simulations show how divergent development can be depending on which sector experiences productivity increases. The agricultural-led regime does industrialize as people are “pushed” out of agriculture. However, this type of development retains large gaps in output per worker across sectors, as well as relatively high fertility. Ultimately, industrialization occurs within the framework of a dual economy and output per person increases only slowly.

The manufacturing-led regime industrializes as well due to the “pull” of higher wages, but this process is not necessarily as complete as in the agricultural-led regime. In contrast, though, the dual economy disappears relatively quickly and fertility falls as well. Ultimately output per capita is significantly higher due to the lower fertility and higher fraction of time spent working.

4.3 Unified growth and the dual economy

Both of the previous sets of simulations took productivity growth to be exogenous. This section incorporates endogenous change in productivity to demonstrate how the dual economy fits within a unified growth framework. The economy will start out in a Malthusian era, with stagnant growth in population and income per capita. Despite the stagnation, agricultural productivity is improving over time and once the economy exits the Malthusian era the dual economy appears. Coincident with the arrival of the dual economy is a surge in fertility and the beginning of the structural transformation. As productivity increases continue the economy enters a modern growth era with low population growth, high income per capita growth, and the disappearance of the dual economy.

To facilitate this, some method of incorporating endogenous productivity growth must be included. Rather than complicate the exposition with a complete micro-economic model of innovation, I adopt a “reduced form” version of endogenous productivity growth similar to Kremer (1993) and Jones (1995a, b). Growth in productivity in the two sectors is described by

$$A^A_{t+1} - A^A_t = \delta_A a_t L_t (A^A_t)^\phi \tag{14}$$

$$A^M_{t+1} - A^M_t = \delta_M (1 - a_t) L_t (A^M_t)^\phi \tag{15}$$

where δ_i is a parameter controlling the speed of innovation, and ϕ measures the returns to scale in innovation. If $\phi = 0$, then the arrival of new innovations is independent of the stock of knowledge, and the growth rate of productivity declines with productivity. If $\phi = 1$ then there are increasing returns to knowledge, and the growth rate depends only on the scale of the sector. To obtain realistic results in the simulations, a value of ϕ is chosen that is less than one, which accords with Jones (1995a, b) evidence on the long-run growth rate of innovation.

Figure 5 shows the time path of several variables over the 600 periods the simulation was run. Panel (A) shows that for nearly the first 400 periods, the economy works exclusively in the agricultural sector. This is due to the fact that initial productivity in that sector is low, and the only way to generate the subsistence requirement is for all individuals to work in agriculture. Productivity is increasing in the agricultural sector during this period, while because $a_t = 1$, there is no productivity growth in the manufacturing sector.

Initially, as panel (B) shows, fertility is low. All individuals work in agriculture, and all individuals spend nearly 100% of their time working to feed themselves. Fertility is non-zero, though, and this begins to generate larger gains in A^A_t over time. Gains in A^A_t free up more time for fertility, and prior to industrialization around period 400 fertility is already increasing. Fertility peaks as agricultural productivity increases to the point where some individuals are freed to work in the manufacturing sector.

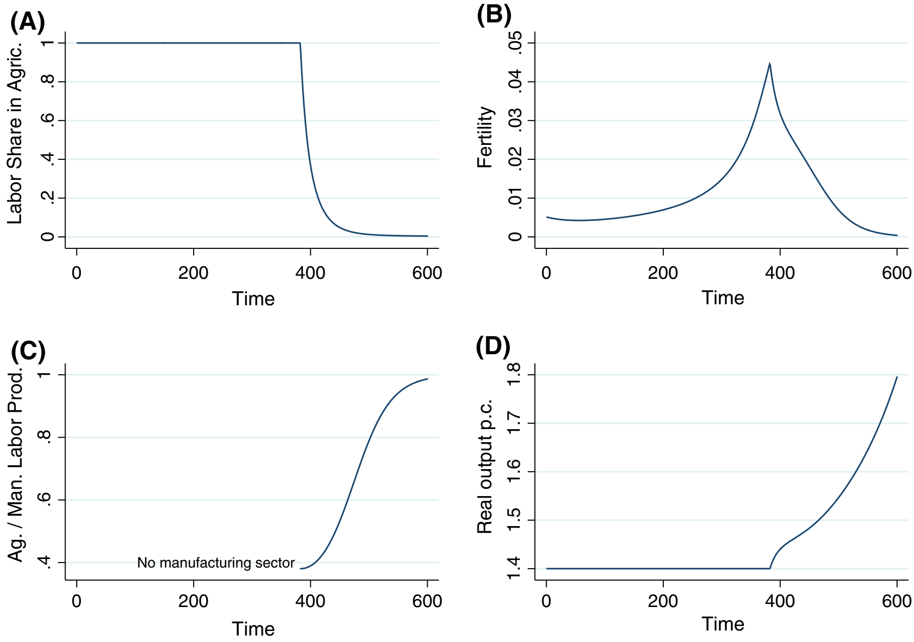

Fig. 5 The dual economy with endogenous growth. *Note*: the panels track the evolution of the economy from the Malthusian era of low fertility, stagnant output per capita, and a large share of workers in the agricultural sector to the modern era of sustained growth, low fertility, and a primarily manufacturing workforce. The dual economy, as denoted by the difference in labor productivity between sectors, arises at the onset of industrialization and then dissipates as the economy develops

Panel (C) shows how the dual economy is evolving during this process. Prior to the release of the first laborers to manufacturing, there is no dual economy because all individuals work in the agricultural sector. Without a manufacturing sector, there can be no difference in relative labor productivity. However, once some individuals enter the manufacturing sector, they immediately begin to spend more time working than their agricultural peers. At the low levels of A_t^M present around period 400 of the simulation, the duality is severe, and agricultural workers produce only 40% of what a manufacturing worker does.

Once manufacturing workers are present, A_t^M begins to increase endogenously, and this generates a flow of workers into manufacturing, while also raising the time allocated to work in both sectors. This causes fertility to decline from its peak, and narrows the gap in output per worker between the sectors. Thus the dual economy endogenously disappears in this unified treatment, and this is driven by the increasing productivity growth of the manufacturing sector.

Ultimately, panel (D) shows how these factors all operate together to generate a Malthusian era of stagnant output per capita that gives way to an era of both high output per capita growth and high fertility around period 400, and finally to the era of modern growth where output per capita growth is high, but fertility has fallen to nearly zero.

The dual economy can be seen as a natural outgrowth of the process of development. Along with the Demographic Transition and the structural transformation, the appearance and gradual dissolution of the dual economy is an integral part of the long-run growth of economies.

5 Conclusion

This papers attempts to show that the dual economy is an integral part of the development of economies. Unlike previous attempts, the model presented here shows how a dual economy can arise endogenously in a model of optimal time allocation.

The mechanism driving the dual economy is that individuals in the agricultural sector internalize the effect of their own work effort on their marginal product, meaning that there is a non-separability in their labor demand and labor supply decisions. Understanding that further work effort lowers the value of their time, they optimally shift towards working less. In contrast, modern sector individuals face a constant wage rate for their time and optimally spend more of their time working.

The model implies that agricultural productivity growth has negative consequences that offset some or all of its initial benefits. Work efforts fall following agricultural productivity changes; the dual economy persists. In contrast, manufacturing productivity increases work efforts in both sectors, resulting in higher output per capita.

Relating the non-market allocation of time to fertility, the model is also able to match the observed sectoral differences in urban/rural fertility rates. This allows an examination of the long-run consequences of different types of productivity growth. Because agricultural productivity growth does not reduce aggregate fertility by as much (and may actually increase it), this type of productivity improvement will actually slow down output per capita growth in the long-run relative to manufacturing improvements.

Ultimately, the persistence of the dual economy in developing countries is not a reflection of embedded institutional or market failures, but rather is a consequence of slow modern sector development. This paper's characterization of the dual economy is in many ways more hopeful than earlier explanations. First, it suggests that one should not interpret differences in output per worker as indicative of differences in welfare between sectors. Second, unlike models relying on exogenous distortions, this model shows that duality does not imply that anything is fundamentally "wrong" with an economy.

Appendix A: Proofs

Proposition 1

Proof To begin, note that utility equalization between sectors implies that $w_t^M s_t^{*M} + V(1 - s_t^{*M}) = p_t^A s_t^{*A} A_t^A F_E + V(1 - s_t^{*A})$, which can be re-arranged to indicate that $p_t^A = [w_t^M s_t^{*M} + V(1 - s_t^{*M}) - V(1 - s_t^{*A})]/[s_t^{*A} A_t^A F_E]$.

Now to see (A), consider what would happen if $s_t^{*M} \leq s_t^{*A}$. From the first order conditions in (11) and (12), this would require that $w_t^M \leq p_t^A A_t^A (F_E + s_t^A F_{EE})$. Given that $F_{EE} < 0$, it would therefore have to be that $w_t^M \leq p_t^A A_t^A F_E$ as well.

Using the characterization of p_t^A from the utility equalization given above, the condition that $w_t^M \leq p_t^A A_t^A F_E$ would require that

$$w_t^M (s_t^{*A} - s_t^{*M}) \leq V(1 - s_t^{*M}) - V(1 - s_t^{*A}). \tag{16}$$

Let $x_t^k = 1 - s_t^k$ for $k \in (A, M)$ and the previous condition would have to be that

$$w_t^M (x_t^{*M} - x_t^{*A}) \leq V(x_t^{*M}) - V(x_t^{*A}). \tag{17}$$

Given the manufacturing first-order condition in (12), this can be restated as the condition that

$$V'(x_t^{*M})(x_t^{*M} - x_t^{*A}) \leq V(x_t^{*M}) - V(x_t^{*A}) \tag{18}$$

which violates the fact that V is a concave function. Therefore, $x_t^{*M} \geq x_t^{*A}$ is false and so is $s_t^{*M} \leq s_t^{*A}$. It follows that $s_t^{*M} > s_t^{*A}$.

Finally, part (B) comes from noting that if $1 - s_t^{*M} < 1 - s_t^{*A}$, then for $U_t^A = U_t^M$ to hold, it must be that $c_t^M > c_t^A$. This implies, given the budget constraint in (10), that $s_t^{*M} w_t^M > s_t^{*A} p_t^A A_t^A F_E\left(\frac{R}{a_t L_t}, s_t\right)$. □

Proposition 2

Proof (A) An increase in A_t^A, given the subsistence requirement in (13), requires that $s_t^A a_t$ falls so that total agricultural supply is equal to demand. Consider what would happen if a_t went up. From (8), this would cause manufacturing wages to rise. In addition, this increase in wages will increase the share of time spent working, s_t^M, which follow from (12). To see this, note that when w_t^M increases, so must $V'(1 - s_t^M)$. As V is concave, an increase in V' requires a decrease in $1 - s_t^M$. A decrease in $1 - s_t^M$ requires s_t^M to increase.

Total utility of manufacturing workers increases as well given their greater resources. Free mobility requires that agricultural utility increases as well, which can only be achieved with an increase in the marginal product of labor, which increases s_t^A. However, if s_t^A goes up, and we have assumed that a_t has increased, then $s_t^A a_t$ goes up, and this contradicts the subsistence requirement that $s_t^A a_t$ goes down. For this subsistence requirement to hold, it must be that a_t falls, which lowers the manufacturing wage, lowering s_t^M, and utility equalization requires that s_t^A falls as well. Both time allocations thus fall.

(B) When A_t^M goes up, this increases w_t^M at every level of a_t. As noted in for part (A), an increase in w_t^M will induce an increase in s_t^M and raise U_t^M. Free mobility requires U_t^A to increase, which is achieved by an increase in the marginal product of labor in agriculture, resulting in s_t^A increasing. With s_t^A higher, it must be that a_t falls so that the subsistence requirement in (13) holds. □

Corollary 2

Proof Consider the effect of a change in any variable x on the utility of an individual in each sector. For an individual in either sector, the total derivative is, applying the Envelope Theorem

$$dU^k = \frac{\partial c^k}{\partial x} dx. \tag{19}$$

The total change in utility following a change in x must be identical in each sector. This means that $\partial c^M / \partial x = \partial c^A / \partial x$. Given the definition of income in each sector, this means that

$$s_t^M \frac{\partial w_t^M}{\partial x} = s_t^A \frac{\partial p_t^A A_t^A F_E}{\partial x}. \tag{20}$$

Given that $s_t^M > s_t^A$ by Proposition 1, it must be the case that $\partial w_t^M / \partial x < \partial p_t^A A_t^A F_E / \partial x$. In words, the change in the marginal product of manufacturing work must be smaller than the change in the marginal product of agricultural work.

The smaller change in the marginal product of work in manufacturing implies, from the first-order condition in (12), a smaller change in the optimal level of time spent working. Following any change in the economy, dx, manufacturing workers will adjust their time less than agricultural workers. Along with the fact that $s_t^M > s_t^A$, any change dx that increases the time allocation to work will therefore shrink the gap between s_t^M and s_t^A. Any change dx that decreases time allocations will increase the gap between s_t^M and s_t^A. Combined with Proposition 2, this means that increases in A_t^M shrink the gap in time allocations, and increases in A_t^A widen the gap in time allocations. $\square$

Appendix B: Simulation

Agricultural production is presumed to be Cobb-Douglas in land and labor effort, as in

$$Y_t^A = A_t^A R^\alpha (s_t^A a_t L_t)^{1-\alpha} \tag{21}$$

where α represents the elasticity of output with respect to land. For simplicity, agricultural workers are assumed to earn all rents from land. As shown in the model, this is not necessary to drive the dual economy result. However, it makes the simulation model tractable.

Given the assumptions regarding property rights, output per person in the agricultural sector can be written as

$$I_t^A = A_t^A \left(\frac{R}{a_t L_t}\right)^\alpha (s_t^A)^{1-\alpha}. \tag{22}$$

Manufacturing output is, for simplicity, assumed to be completely linear in labor effort,

$$Y_t^M = A_t^M s_t^M (1 - a_t) L_t. \tag{23}$$

The wage rate per unit of labor effort is simply $w_t^M = A_t^M$ in this case, and there is no effect of $(1 - a_t)$ on the wage rate.

Individual utility for someone in sector i is assumed to have the following form

$$U_t^i = c_t^{Mi} + (n_t^i)^{1-\alpha} \tag{24}$$

where fertility shows diminishing marginal utility, and the assumption that the exponent is equal to $1 - \alpha$ is made so that a clear analytical solution can be obtained.

Fertility can be written as $n_t^i = \overline{n}(1 - s_t^i)$. Using the budget constraint in (10) we can write utility in the two sectors as

$$U_t^A = p_t^A A_t^A \left(\frac{R}{a_t L_t}\right)^\alpha (s_t^A)^{1-\alpha} - p_t^A \overline{b} + \hat{n}(1 - s_t^A)^{1-\alpha} \tag{25}$$

$$U_t^M = A_t^M s_t^M - p_t^A \overline{b} + \hat{n}(1 - s_t^M)^{1-\alpha} \tag{26}$$

where $\hat{n} = \overline{n}^{1-\alpha}$.

The final condition is the subsistence constraint, as is (13), which given the agricultural production function in (21) yields

$$\overline{b} L_t = A_t^A R^\alpha (s_t^A a_t L_t)^{1-\alpha}. \tag{27}$$

Now applying the concept of equilibrium from Definition 1, we must have individuals maximizing the utility functions in (26), while free mobility ensures that $U_t^A = U_t^M$, and subsistence requirements are met as in (27). First, solving for the optimal time allocation in the manufacturing sector yields

$$s_t^{*M} = 1 - \left(\frac{\hat{n}(1-\alpha)}{A_t^M}\right)^{1/\alpha} \tag{28}$$

and knowing this, define the following term

$$\hat{U}_t \equiv A_t^M s_t^{*M} + \hat{n}(1 - s_t^{*M})^{1-\alpha} \tag{29}$$

which reflects the utility of manufacturing individuals if their subsistence requirement were zero.

Knowing this, the rest of the equilibrium can be found. First, maximizing U_t^A over s_t^A yields the following result

$$\frac{s_t^{*A}}{1 - s_t^{*A}} = \left(\frac{p_t^A A_t^A}{\hat{n}}\left(\frac{R}{a_t L_t}\right)^{\alpha}\right)^{1/\alpha} \tag{30}$$

which when inserted back into U_t^A and used with the free mobility condition and the definition of $\hat{U}$ gives

$$s_t^{*A} = 1 - \left(\frac{\hat{n}}{\hat{U}}\right)^{1/\alpha}. \tag{31}$$

Given s_t^{*A}, the optimal share of labor in agriculture is obtained from the subsistence constraint in (27) which yields

$$a_t^* = \left(\frac{\overline{b} L^{\alpha}}{A_t^A R^{\alpha} (s_t^{*A})^{1-\alpha}}\right)^{1/(1-\alpha)} \tag{32}$$

and finally the price of agricultural goods can be solved from (30) giving

$$p_t^{*A} = \frac{\hat{n}}{A_t^A}\left(\frac{a_t^* L_t}{R}\frac{s_t^{*A}}{1 - s_t^{*A}}.\right)^{\alpha} \tag{33}$$

Given s_t^{*A}, s_t^{*M}, a_t^*, and p_t^{*A}, we have all the information available to solve for other variables of interest within a given period. Specifically, output in each sector follows directly from (21) and (23). Output per capita is found simply by dividing their total by L_t. The relative output of agricultural workers to manufacturing workers can be directly backed out from the production functions.

The dynamic effects of changes in productivity depend entirely in this set-up on the fertility consequences. Population evolves as follows

$$L_{t+1} = (1 + n_t)L_t \tag{34}$$

where

$$n_t = a_t \overline{n}(1 - s_t^{*A}) + (1 - a_t)\overline{n}(1 - s_t^{*M}). \tag{35}$$

As population increases, this acts much like a *decrease* in agricultural productivity, raising agricultural prices and inducing more people to stay in agriculture.

The parameters in column (1) of Table 1 yield initial values of $s_t^{*M} = 0.54$ and $s_t^{*A} = 0.17$ for the baseline model, which results in output per worker in the agricultural sector being only 38% of that in the manufacturing. To meet subsistence, $a_0 = 0.999$.

Table 1 Simulation parameter values

Parameter	(1) Base model	(2) Unified model
Land (R)	10	1
Land share (α)	0.3	0.3
Subsistence req. ($\overline{b}$)	1.4	1.4
Fertility constant ($\overline{n}$)	0.054	0.54
Initial population (L_0)	1	1
Initial agric. productivity (A_0^A)	2.4	1.5
Initial manuf. productivity (A_0^M)	0.115	0.115

Once endogenous productivity growth is introduced, the parameters of the innovation equations in (15) must be specified as well. For the simulations presented in this paper, the value of $\phi = 0.99$, while $\delta_A = 0.001$ and $\delta_M = 0.00001$. These values were chosen by trial and error so as to yield maximum growth in productivity in any given period below 5%.

In addition to the endogenous productivity parameters, the baseline model is modified in the manner indicated in column (2) of Table 1. These act mainly to reduce the agricultural output of the economy, meaning that subsistence cannot be achieved with fewer than 100% of individuals in the agricultural sector. With $a_t = 1$, the necessary work effort in agriculture is

$$s_t^A = \left(\frac{\overline{a} L_t^{\alpha}}{A_t^A R_t^{\alpha}} \right)^{1/(1-\alpha)}. \tag{36}$$

Once A_t^A is sufficiently large so that the subsistence requirement can be met with less than 100% of the individuals in agriculture, then the equilibrium is calculated as before.

References

Banerjee, A., & Duflo, E. (2005). Growth theory through the lens of development economics. In P. Aghion & S. Durlauf (Eds.), *Handbook of economic growth*. New York: North Holland.

Basu, K. (1997). *Analytical development economics: The less developed economy revisited*. Cambridge, MA: MIT Press.

Benjamin, D. (1992). Household composition, labor markets, and labor demand: Testing for separation in agricultural household models. *Econometrica, 60*(2), 287–322.

Bertrand, T., & Squire, L. (1980). The relevance of the dual economy: A case study of Thailand. *Oxford Economic Papers, 40*, 55–73.

Binswanger, H., & Rosenzweig, M. (1986). Behavioral and material determinants of production relations in agriculture. *Journal of Development Studies, 22*, 503–539.

Caselli, F., & Coleman, J. W. (2001). The US structural transformation and regional convergence: A re-interpretation. *Journal of Political Economy, 109*(31), 584–616.

Caselli, F. (2005). Accounting for cross-country income differences. In A. Phillipe & S. Durlauf (Eds.), *Handbook of economic growth*. New York: North Holland.

Chanda, A., & Dalgaard, C.-J. (2008). Dual economies and international total factor productivity differences: Channelling the impact from institutions, trade, and geography. *Economica, 75*, 629–661.

Chenery, H., & Syrquin, M. (1975). *Patterns of development, 1950–1970*. London: Oxford University Press.

Chenery, H., Robinson, S., & Syrquin, M. (1986). *Industrialization and growth: A comparative study*. New York: Oxford University Press.

Clark, G. (1999). Too much revolution: Agriculture in the industrial revolution, 1700–1860. In J. Mokyr (Ed.), *The british industrial revolution* (pp. 206–240). Oxford, UK: Westview Press.
Cordoba, J. C., & Ripoll, M. (2008). *Agriculture, aggregation, and development accounting*. Working Paper.
Dahan, M., & Tsiddon, D. (1998). Demographic transition, income distribution and economic growth. *Journal of Economic Growth, 3*, 29–52.
David, P. A. (2005). *Real income and economic welfare growth in the early republic*. Stanford University Working Paper.
de la Croix, D., & Doepke, M. (2003). Inequality and growth: Why differential fertility matters. *American Economic Review, 93*, 1091–1113.
de Vries, J. (1994). The industrial revolution and the industrious revolution. *Journal of Economic History, 54*, 249–270.
Eastwood, R., Lipton, M., & Newell, A. (2008). Farm size. In R. Evenson & P. Pingali (Eds.), *Handbook of agricultural economics* (Vol. IV). North Holland: New York.
Eswaran, M., & Kotwal, A. (1985). A theory of contractual structure in agriculture. *American Economic Review, 75*, 352–367.
Findley, S., & Orr, A. (1978). *Patterns of urban-rural fertility differentials in developing countries: A suggested framework*. Washington, DC: Center for Advanced Studies.
Galor, O. (2005). From stagnation to growth: Unified growth theory. In A. Phillipe & S. Durlauf (Eds.), *Handbook of economic growth*. New York: North Holland.
Galor, O., Moav, O., & Vollrath, D. (2009). Inequality in land ownership, the emergence of human capital promoting institutions and the great divergence. *Review of Economic Studies, 76*, 143–179.
Galor, O., & Mountford, A. (2008). Trading population for productivity: Theory and evidence. *Review of Economic Studies, 75*, 1143–1179.
Gollin, D., Parente, S., & Rogerson, R., (2002). The role of agriculture in development. *American Economic Reivew Papers and Proceedings*.
Gollin, D., Parente, S., & Rogerson, R. (2004). Farm work, home work and international productivity differences. *Review of Economic Dynamics, 7*, 827–850.
Goodfriend, M., & McDermott, J. (1995). Early development. *American Economic Review, 85*(1), 116–133.
Grabill, W. H., Kiser, C. V., & Whelpton, P. K. (1958). *The fertility of American women*. New York: John Wiley and Sons, Inc.
Hansen, G., & Prescott, E. C. (2002). Malthus to solow. *American Economic Review, 92*(4), 1205–1217.
Harris, J. R., & Todaro, M. P. (1970). Migration, unemployment and development: A two-sector analysis. *American Economic Review, 60*, 126–142.
Hayashi, F., & Prescott, E. (2008). The depressing effect of agricultural institutions on the prewar japanese economy. *Journal of Political Economy, 116*(4), 573–632.
Jones, C. I. (1995a). R&D-based models of economic growth. *Journal of Political Economy, 103*(4), 759–784.
Jones, C. I. (1995b). Time series tests of endogenous growth models. *Quarterly Journal of Economics, 110*(2), 495–525.
Jorgenson, D. W. (1961). The development of a dual economy. *Economic Journal, 71*(282), 309–334.
Jorgenson, D. W., & Gollop, F. M. (1992). Productivity growth in U.S. agriculture: A post-war perspective. *American Journal of Agricultural Economics, 74*(3), 745–750.
Knodel, J. E. (1974). *The decline of fertility in Germany, 1871–1939*. Princeton, NJ: Princeton University Press.
Kogel, T., & Prskawetz, A. (2001). Agricultural productivity growth and escape from the malthusian trap. *Journal of Economic Growth, 6*, 337–357.
Kongasmut, P., Rebelo, S., & Xie, D. (2001). Beyond balanced growth. *Review of Economic Studies, 68*(4), 869–882.
Kremer, M. (1993). Population growth and technological change: One million B.C. to 1990. *Quarterly Journal of Economics, 108*, 681–716.
Kuznets, S. (1966). *Modern economic growth*. New Haven: Yale University Press.
Kuznets, S. (1974). Rural-Urban differences in fertility: An international comparison. *Proceedings of the American Philosophical Society, 118*(1), 1–29.
Laitner, J. (2000). Structural change and economic growth. *Review of Economic Studies, 67*, 545–561.
Lesthaeghe, R. J. (1977). *The decline of Belgian fertility, 1800–1970*. Princeton, NJ: Princeton University Press.
Lewis, W. A. (1954). Economic development with unlimited supplies of labor. *The Manchester School, 22*, 139–191.
Livi-Bacci, M. (1977). *A history of Italian fertility during the last two centuries*. Princeton, NJ: Princeton University Press.

Matsuyama, K. (1992). Agricultural productivity, comparative advantage, and economic growth. *Journal of Economic Theory, 58*, 317–334.

Mourmouras, A., & Rangazas, P. (2007). Reconciling Kuznets and Habbakuk in a Unified Growth Theory. IUPUI Working Paper 2007-04.

Parente, S., Rogerson, R., & Wright, R. (2000). Homework in development economics: Household production and the wealth of nations. *Journal of Political Economy, 108*(4), 680–687.

Proto, E. (2007). Land and the transition from a dual to a modern economy. *Journal of Development Economics, 83*, 88–108.

Ranis, G. (1988). Analytics of development: Dualism. In H. Chenery & T. N. Srinivasan (Eds.), *Handbook of development economics* (Vol. I). New York: North Holland.

Ranis, G., & Fei, J. C. H. (1961). A theory of economic development. *American Economic Review, 51*(4), 533–565.

Restuccia, D., Yang, D. T., & Zhu, X. (2008). Agriculture and aggregate productivity: A quantitative cross-country analysis. *Journal of Monetary Economics, 55*, 234–250.

Rosenzweig, M. (1988). Labor markets in low-income countries. In H. Chenery & T. N. Srinivasan (Eds.), *Handbook of development economics* (Vol. I). North Holland: New York.

Roumasset, J. (1995). The nature of the agricultural firm. *Journal of Economic Behavior and Organization, 26*, 161–177.

Sharlin, A. (1986). Urban-rural differences in fertility in Europe during the demographic transition. In A. J. Coale & S. C. Watkins (Eds.), *The decline of fertility in europe*. Princeton, NJ: Princeton University Press.

Stiglitz, J. E. (1974). Alternative theories of wage determination and unemployment in LDCs: The labour turnover model. *Quarterly Journal of Economics, 88*, 194–227.

Strulik, H., & Weisdorf, J. (2008). Population, food, and knowledge: A simple unified growth theory. *Journal of Economic Growth, 13*(3), 195–216.

Tamura, R. (2002). Human capital and the switch from agriculture and industry. *Journal of Economic Dynamics and Control, 27*, 207–242.

Temple, J. (2005). Dual economy models: A primer for growth economists. *The Manchester School, 73*(4), 435–478.

Temple, J., & Woessmann, L. (2006). Dualism and cross-country growth regressions. *Journal of Economic Growth, 11*(3), 187–228.

Vollrath, D. (2009). How important are dual economy effects for aggregate productivity? *Journal of Development Economics, 88*, 325-334.

Weisdorf, J. (2006). From domestic manufacture to industrial revolution: Long-run growth and agricultural development. *Oxford Economic Papers, 58*(2), 264–287.