The Speed of Convergence

Background

In the canonical Solow model, differences in growth rates are due to differences in initial conditions. But in the long run, growth rate differences would disappear as all economies reach their steady state. This appears to be true for OECD economies, but not for the world as a whole.
Barro and Sala-i-Martin (1992) looked within the United States, and found strong evidence of convergence. This figure shows the plot of the average growth rate from 1880 to 1988 against the initial level of income per capita in 1880.

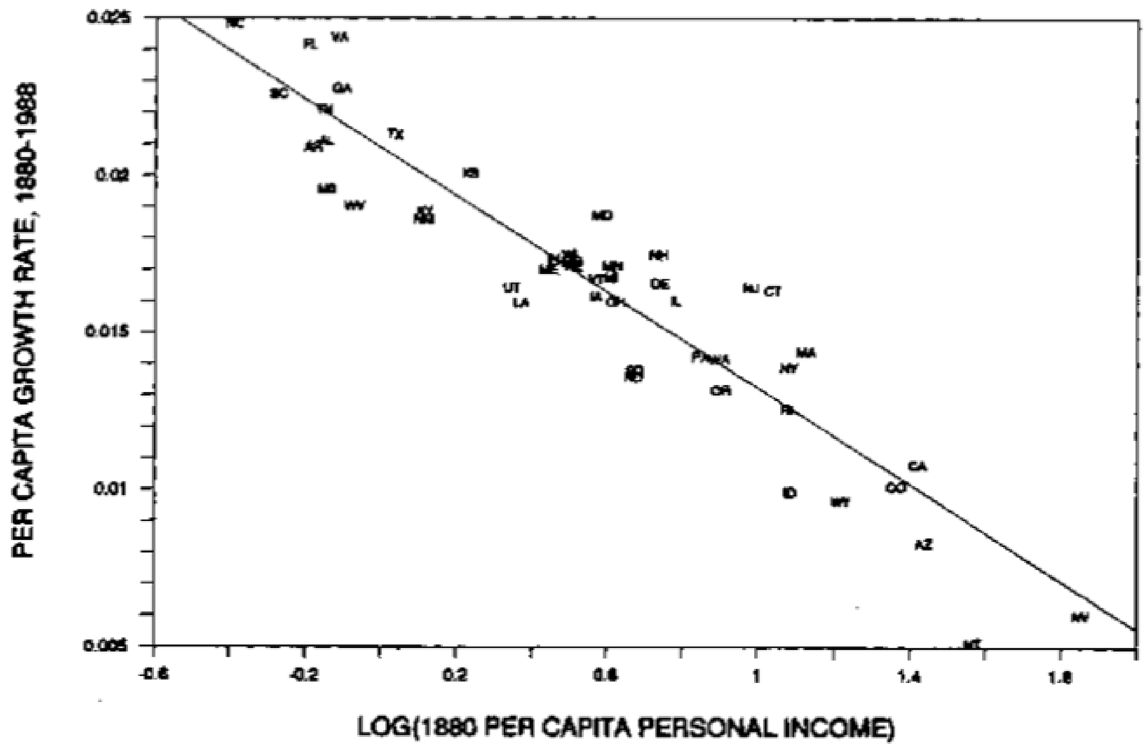

FIG. 1.—Growth rate from 1880 to 1988 vs. 1880 per capita income

Figure 1: Convergence in US States

This kind of relationship holds within a number of other countries that Sala-i-Martin examined in subsequent work: Japanese prefectures, Canadian provinces, etc.. So while the Solow model might be too limited to describe cross-country growth, it appears to do a good job of describing within-country growth.
One thing this convergence plot doesn't tell you is *how long* does it take the poor places to catch up, which you are going to try and figure out.

Project

Take the basic Solow model, with population growth, productivity growth, and depreciation. I want you to do two things:

1. Derive a closed form solution for output per capita in any period, y_t, in terms of only the initial level of output per capita, y_0, and the exogenous parameters of the Solow model.
2. Find how long it takes for a country to close half the distance between its initial level of output per efficiency unit, y_0, and it's steady state level of output per efficiency unit, y^*.

The hint I will give you is that you should think about doing the analysis in terms of a variable, call it z_t, that is $z_t = y_t^{1-\alpha}$. Figure out the closed form solution for z_t, and then you can back out the solution for y_t. The other piece of mathematical information you need is that for a given difference equation $x_t = ax_{t-1} + b$, the solution for x_t in terms of x_0 is given by

$$x_t = a^t x_0 + (1 - a^t)\frac{b}{1-a}.$$

Note that x_t at any given t is a weighted sum of the initial value, x_0, and the steady state value $b/(1-a)$. All that changes over time is the weight.

Rules

You should work on this project alone, but we will work through it in stages during class.