

# The Decline in Labor's Share of Output

## Background

With a standard Cobb-Douglas production function, and functioning markets for labor and capital, we'd expect that the share of output paid to labor,  $wL/Y$ , to be constant over time. This was one of Kaldor's set of facts about modern economies (along with stable growth rates and stable rates of return to capital).

However, recent evidence shows that labor's share of output has been declining over the last few decades. The figure here is from Karabarbounis and Neiman (2013), and shows the decline for several large economies. Their paper establishes that this is a regular feature of most industrialized economies.

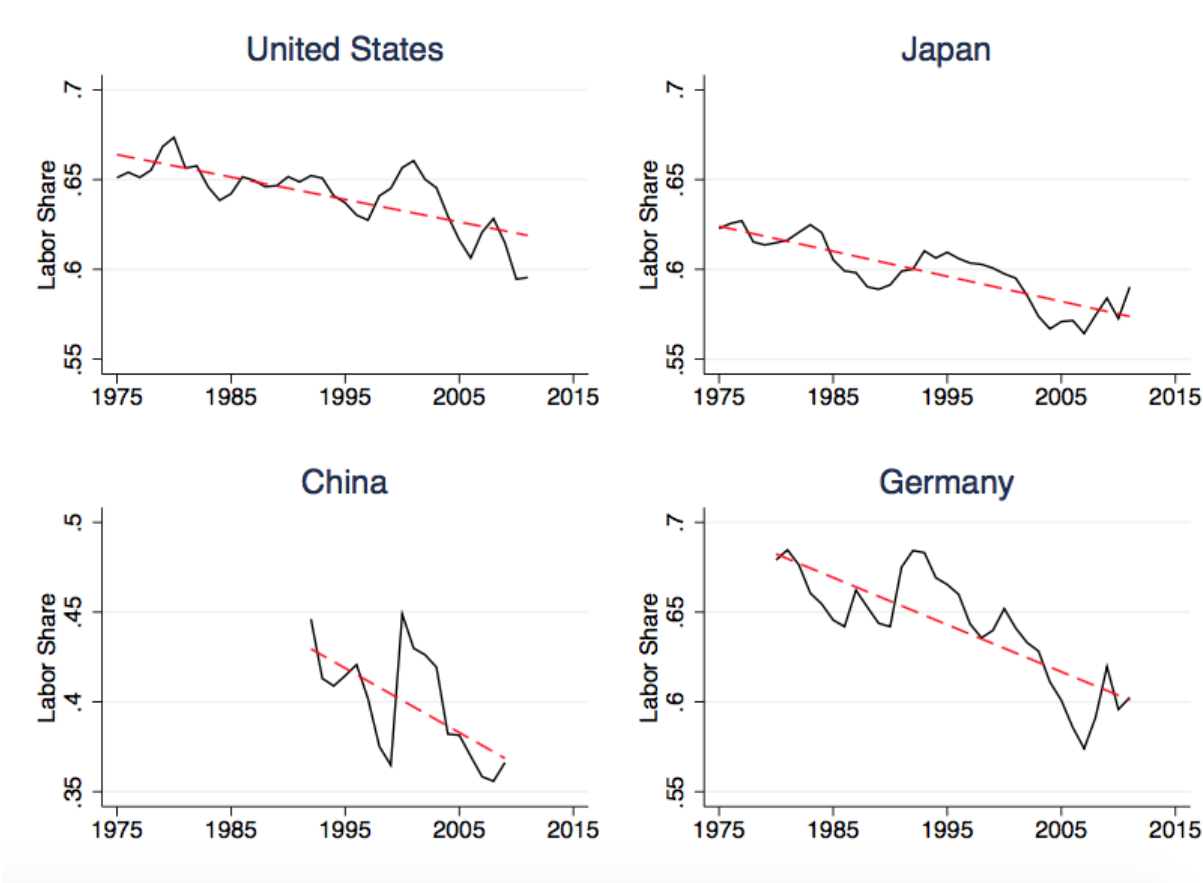


Figure 1: The Decline in Labor's Share

## Project

As our standard model with Cobb-Douglas production does not accommodate this fact, your goal is to write down a model of economic growth, similar to the Solow model, that allows for changes in the share of output paid to labor (and hence capital) over time.

To do this, it will be best to think about a flexible production function called a Constant Elasticity of Substitution (CES) function, which can be written as

$$Y = \left( \alpha K^{\frac{\theta-1}{\theta}} + (1-\alpha)L^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}.$$

$\theta$  is the elasticity of substitution between K and L. If  $\theta$  is zero, then (in the limit) this function reduces to  $Y = \min(K, L)$ , and capital and labor are perfect complements. If  $\theta$  is one, then (in the limit) this function reduces to a Cobb-Douglas with coefficients  $\alpha$  and  $1 - \alpha$  on capital and labor. If  $\theta$  goes to infinity, then (in the limit) production is  $Y = \alpha K + (1 - \alpha)L$ , or capital and labor are perfect substitutes.

Write down a model of growth that incorporates the following elements

1. CES production for firms
2. Capital accumulation using a constant savings rate
3. Competitive firms that are price-takers
4. Population growth at the rate  $n$

With that model, you should do the following

1. Derive the steady state capital/labor ratio, and output per worker, if possible
2. Derive labor's share of output
3. Show conditions under which labor's share of output would fall over time
4. Derive the steady state labor share of output if the conditions in 3 hold

## Rules

You should work on this project alone. You will have to show me, either in written form or on the board, the model you set up, and explain to me how it works.