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The Elasticity of Aggregate Output with Respect to Capital and Labor

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ABSTRACT

I estimate the elasticity of GDP with respect to aggregate capital and labor using industry-specific cost structures and without assuming an aggregate production function exists. Different assumptions about industry capital costs provide upper and lower bounds for the elasticities. In the U.S. from 1948-1995 the capital elasticity bounds were 0.18-0.33, and rose to 0.21-0.39 in 1995-2018. Excluding housing and government or de-capitalizing intellectual property lowers the capital elasticity bounds. Elasticities are similar in other OECD countries from 2005-2015. Based on these boundaries for the capital elasticity, common estimates of total factor productivity growth represent a lower bound.

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1 Introduction

One of the most common assumptions made within economics is that “alpha equals one-third”, referring to the capital elasticity α in a Cobb-Douglas aggregate production function $Y = K^\alpha L^{1-\alpha}$. This rule of thumb is derived from an observation that labor’s share of GDP is around two-thirds, implying $1 - \alpha \approx 2/3$, and hence that $\alpha \approx 1/3$. Not only has a recent literature (reviewed below) documented that labor’s share of GDP has fallen in the last few decades, but as Hall (1988, 1990) noted this rule of thumb only works if one assumes there are zero economic profits and hence labor’s GDP share is equal to it’s elasticity. Moreover, the rule of thumb relies on the existence of an aggregate production function and commonly assumes that α is constant over time.

In recent work Baqaee and Farhi (2019, 2020) show that one can calculate a meaningful elasticity of GDP with respect to capital, $d \ln Y / d \ln K$, without having to rely on any of the assumptions embedded in the rule of thumb. In particular, their theory shows how to calculate the aggregate elasticity from disaggregated units (e.g. industries) with rich input/output relationships and arbitrary unit-level distortions (e.g. markups). There is no need to assume an aggregate production function exists, that profits are zero, or that the elasticity remains constant over time. Their structure requires only market-clearing and cost-minimization.

In this paper I apply their theory to calculate the annual elasticity of GDP with respect to capital and labor using disaggregated data on industries and their input-output relationships, allowing for arbitrary markups at the industry level. I do this for the United States over the period 1948-2018, and make a comparison across a set of 20 OECD countries for the period 2005-2015.

The theory in Baqaee and Farhi (2019, 2020) does not eliminate the well-known problem of separating capital costs from economic profits in national accounts data. Because of this, what I present here are plausible bounds on the capital and labor elasticities based on different assumptions regarding capital costs. An upper bound for the capital elasticity is established by assuming zero economic profits in all industries, such that all value-added not used for labor compensation is paid to capital, as in the rule of thumb. This represents one extreme of the capital elasticity, and true values are likely lower. A lower bound for the capital elasticity is found by assuming capital costs are equal to depreciation, as industries pay at least this amount in capital costs.¹ Bounds for the labor elasticity are simply one minus the capital elasticity, so the no-profit assumption represents a lower bound, and the depreciation assumption an upper bound, for the labor elasticity.

My baseline bounds for the capital elasticity in the U.S. can be seen in Figure 1. Between 1948 and 1995, the elasticity of GDP with respect to capital was in a range of 0.18-0.33, with 1/3 forming an upper bound except for the 1970s. After 1995 the range shifted up, and from 1995-2018 the elasticity with respect to capital was 0.21-0.39. The bounds for the labor elasticity are the mirror

¹There are measurement issues with labor costs as well, in particular with the treatment of proprietors income (Gollin, 2002; Gomme and Rupert, 2004; Elsby, Hobijn and Sahin, 2013). In practice the treatment of proprietors income generates little variation in the estimated elasticities.

image of these.

Because of the issues with measuring capital costs I cannot give a precise estimate of the capital elasticity or labor elasticity. To the extent that there are markups present in the economy, the actual capital elasticity must lie below the upper bound (and hence the labor elasticity above its lower bound). Given recent evidence that markups were above one throughout the period 1948-2018 (Barkai, 2017; Edmond, Midrigan and Xu, 2018; De Loecker and Eeckhout, 2017; Gutiérrez and Philippon, 2016; Basu, 2019) this implies that the capital elasticity was *below* one-third from 1948-1995. After 1995, it becomes less clear. Higher markups in that period imply the capital elasticity was below the upper bound, but at the same time the upper bound on the capital elasticity rose. A value of one-third for the capital elasticity after 1995 is plausible, but in no way certain.

In addition to the bounding estimates, I explore two alternative ways of imputing the capital costs that go into the elasticity calculations. The first alternative is based on a user cost formula (Hall and Jorgenson, 1967) as in Barkai (2017) and Rognlie (2015). The elasticity estimates based on user costs of capital fluctuate from 1948-2018 and show an upward drift, but for the most part lie within the bounds. There are exceptions that imply periods of widespread negative economic profits.² As a second alternative, I use investment spending by industries to estimate their capital costs. The capital elasticity based on these costs shows a smaller upward trend, and stays everywhere within the bounds. On average, the capital elasticity in this case is around 0.27 after 1960, after beginning close to 0.21 in 1948.

The bounds and alternative estimates of the capital elasticity all show an upward trend to some extent. I use an Olley and Pakes (1996) decomposition of the elasticity estimates to investigate the source of this. On average, the capital elasticity across all industries drove the aggregate trend, and changes in the composition of GDP by industry did not. There is evidence that larger industries tend to have higher capital elasticities, but that relationship did not strengthen or weaken over time.

Beyond these baseline results, the bounds on the elasticities depend on the scope of economic activity included. In particular, if I narrow my focus to the private business sector (excluding government and owner-occupied housing) then the estimated capital elasticity is lower than in the baseline. The capital elasticity in the private business sector is 0.09-0.28 from 1948-1995, and 0.16-0.33 from 1995-2018, meaning it is always below one-third.

In a different exercise, I de-capitalize intellectual property (IP) from the national accounts data as in Koh, Santaaulàlia-Llopis and Zheng (2020). This shifts down the estimates of the capital elasticity as well. From 1948-1995 the capital elasticity is in the range 0.14-0.32, and 0.17-0.35 from 1995-2018. The adjustment is not as dramatic as with the private business sector, but capitalization of IP does appear to account for several points of the overall increase in the capital elasticity over

²User cost based estimates of the capital elasticity in 1975-1989 often lie above the upper bound, implying negative economic profits. This is driven by assumptions made regarding expected inflation in the user cost formula. I discuss this in Section 4 when covering the results in more detail.

time.

The last main result of the paper is the comparison across 20 OECD countries from 2005-2015. Here I find that the bounds in all countries overlap to a large extent. The level of the capital elasticity across OECD countries is slightly higher (and the labor elasticity slightly lower) than in my U.S. baseline, but this is partly due to a different treatment of proprietors income in the OECD due to data limitations. Assessing the U.S. under the same terms as the OECD, the overlap in elasticities is strong.

This paper is a complement to the growing literature on the distribution of GDP across factors.³ It shares with that literature the same measurement issues surrounding proprietors income and capital costs. Recent work on “factorless income” by Karabarbounis and Neiman (2019) is perhaps the closest methodological analogue to this paper, in that those authors explore a range of plausible approaches for dealing with this factorless income, although at an aggregate level. The bounds I find for the elasticities are calculated either assuming that all factorless income is attributed to capital (the zero profit bound) or that it all factorless income represents economic profits (the depreciation cost bound).⁴

Despite the similarities in approach with the literature on factor shares of GDP, my findings are distinct. Elasticities and factor shares are not identical due both to market power and the input-output structure of the economy (Baqae and Farhi, 2019, 2020). For this reason a factor’s share of GDP does not necessarily inform us about the elasticity of GDP with respect to that factor.

One of the uses of the elasticities I estimate is as an input into other macroeconomic studies. Any research that is interested in how the supply of factors of production (e.g. savings/consumption decisions, labor force participation, demographic change, international finance) will affect the economy can use these elasticities without having to specify an entire production structure that incorporates market power or rich input-output relationships. As Baqae and Farhi (2019, 2020) show, these elasticities already embed those features. As I can only provide reasonable bounds for the elasticities, this provides a range of values that factor supply models could use to evaluate their results. One caution is that the elasticities are first-order approximations, and so any dramatic changes in factor supplies would have to account for second-order effects (Baqae and Farhi, 2018).

As different application is to growth accounting, which depends on elasticity estimates as part

³Azmat, Manning and Reenen (2012); Bentolila and Saint-Paul (2003); Estrada and Valdeolivas (2014); Harrison (2005); Jaumotte and Tytell (2007); Guscina (2006); Karabarbounis and Neiman (2014); Dao et al. (2017) all document a decline in labor’s share of GDP in the last few decades, across countries and industries. This was contemporaneous with a decline in capital’s share of GDP (Barkai, 2017; Rognlie, 2015). Incorporating the lessons in Gollin (2002) regarding proprietors income does not appear to change that conclusion (Gomme and Rupert, 2004; Elsby, Hobijn and Sahin, 2013). The decline in labor’s share has been tied to a fall in the price of new capital (Karabarbounis and Neiman, 2014), but more recent research suggests it may be an artifact of capitalizing intellectual property (Koh, Santaaulàlia-Llopis and Zheng, 2020) or the reporting of income (Smith et al., 2019).

⁴Factorless income as a share of value-added is larger in the industry-level data than in the aggregate because I do not have information on some rental costs that are reported at the aggregate level.

of the calculation of growth in total factor productivity (TFP). Typical accounting exercises by the Bureau of Labor Statistics, as well as extensions to incorporate utilization rates (Kimball, Fernald and Basu, 2006; Fernald, 2014), use the factor share of labor to find the labor elasticity and one minus that share to find the capital elasticity, which is equivalent to the no-profits bound on the elasticities. Based on my results, that overstates the capital elasticity and understates the labor elasticity. In the last section of this paper I use the bounds on the elasticities to create bounds for the growth of TFP in the U.S. from 1948-2018. The typical no-profit assumption represents a *lower* bound for the growth rate of TFP, which averages about 1.3% per year 1948-2018. The growth rate of TFP may have been up to 1.6% per year, depending on the choice of assumption used to find the capital and labor elasticities. The level of TFP in 2018 may be up to 20% higher than what the typical calculation finds.

The paper proceeds as follows. Section 2 presents the theoretical framework of Baqaee and Farhi (2019, 2020) I use to calculate the elasticities, and Section 3 discusses the data sources and major measurement issues. In Section 4 I present the baseline results on the bounds for the elasticities, as well as the alternatives based on investment and user cost assumptions. Section 5 evaluates trends in the elasticity estimates and their relationship to aggregate ratios of costs to GDP. In Section 6 I explore how the elasticities change depending on the scope of economic activity. Section 7 compares elasticities across the OECD, and Section 8 performs the growth accounting exercises. Section 9 concludes.

2 Theoretical background

What I present in this section is a simplified version of Baqaee and Farhi (2019, 2020) to highlight only the theory that I use. Full proofs and deeper explanations can be found in their papers. In the interest of space I use the abbreviation “BF” to refer to those authors in this section.

The economy consists of J industries, and each industry uses intermediate inputs from other industries (and possibly itself), as well as the factors of production capital (K) and labor (L). There can be any arbitrary number of factors of production, and I use capital and labor here only for simplicity. BF’s theory is also “nestable” in that each industry could have an arbitrary number of sub-industries or firms inside of it. I am speaking of industries here solely because this is the level of detail I have available in the data.

The gross production function of any industry has constant returns with respect to intermediates and the factors of production, but no other structure is imposed. Each industry is assumed to be cost-minimizing, and each industry charges a price for their output that is a markup over the marginal cost. For my purposes I will not need to know the price or the markup. It will be sufficient to speak only about the costs faced by each industry.

Everything I present in this section holds for a given period t . To avoid needless notation, for

the remainder of this section I will not use the t subscript.

To begin, for industry i let the costs of intermediate good purchased from industry j be denoted as $COST_{ij}$. The sum of costs accounted for by intermediate goods purchased by industry i from all other J industries are then

$$COST_{iM} = \sum_{j=1}^J COST_{ij} \quad (1)$$

where the letter M is used to denote that this represents intermediate good costs only.

The capital costs faced by industry i will be denoted $COST_{iK}$, and the labor costs of the same industry will be denoted $COST_{iL}$. Combined with the intermediate good summation above, this means that total costs for industry i are

$$COST_i = COST_{iM} + COST_{iK} + COST_{iL}. \quad (2)$$

Using these total costs one can calculate cost shares, which will be the most relevant piece of information for calculating the aggregate elasticities in the end. For industry i the share of total costs accounted for by intermediate purchases from industry j is defined as

$$\lambda_{ij} = \frac{COST_{ij}}{COST_i}. \quad (3)$$

In a similar manner, for industry i the share of total costs accounted for by capital and labor, respectively, are

$$\lambda_{iK} = \frac{COST_{iK}}{COST_i} \quad (4)$$

$$\lambda_{iL} = \frac{COST_{iL}}{COST_i}. \quad (5)$$

These cost shares can be used to build a modified input-output matrix that BF show can be used to calculate the aggregate elasticities with respect to labor and capital. The insight from BF is that one can treat the factors of production as “industries” that serve as an input to other industries, but which purchase no intermediates from other industries. By including them in an expanded I/O matrix they show how this can be used to solve for the elasticity of aggregate output with respect to those factors.

Let Λ be a $J + 2$ by $J + 2$ matrix, which has J rows/columns from the individual industries, and two additional rows/columns, one each for the capital and labor “industries”. Each row of Λ is associated with an industry i , and the entries show the cost share of industry i coming from the industry j represented in the columns. The cost shares of capital and labor for industry i are

included as the final two columns of each row.

The last two *rows* of Λ capture the cost structure of the capital and labor, and they are “dummy” rows in the sense that each entry is a zero. The capital and labor industries do not employ any intermediate goods themselves, nor do they hire labor or capital.

It is easiest to understand the structure of Λ by examining it,

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1J} & \lambda_{1K} & \lambda_{1L} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2J} & \lambda_{2K} & \lambda_{2L} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \lambda_{J1} & \lambda_{J2} & \cdots & \lambda_{JJ} & \lambda_{JK} & \lambda_{JL} \\ 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 \end{bmatrix} \quad (6)$$

The top-left $J \times J$ block of this matrix is a standard input-output matrix. The final two columns represent the capital and labor costs, respectively, for each industry. Note that the sum across a row is equal to one for each of the first J rows, simply indicating that the matrix accounts for the costs facing industry i . The final two rows of the matrix are the dummy rows for capital (second to last) and labor (last row).

A calculation of aggregate elasticities requires one final piece of information. Let va_j be the value-added of industry j , and $VA = \sum_{j=1}^J va_j$ be total value-added. Then let the value-added share of industry j be denoted by

$$\gamma_j = \frac{va_j}{VA}. \quad (7)$$

Last, create a $J + 2$ by one vector Γ which has γ_j in row j for the first J rows, and zeroes in the last two rows. Those last two rows represent the value-added shares of the capital and labor industries, which are zero as those two factors are used solely as inputs by other industries. The structure of Γ is

$$\Gamma' = \begin{bmatrix} \gamma_1 & \gamma_2 & \cdots & \gamma_J & 0 & 0 \end{bmatrix} \quad (8)$$

Given this information, one can calculate the vector of what BF call “cost-based Domar weights”, E , for each industry.

$$E = \Gamma'(I - \Lambda)^{-1} \quad (9)$$

where I is a $J + 2$ square identity matrix, and $(I - \Lambda)^{-1}$ is the Leontief inverse matrix of the expanded input-output matrix.

The structure of E is as follows,

$$E = \begin{bmatrix} \epsilon_1 & \epsilon_2 & \cdots & \epsilon_J & \epsilon_K & \epsilon_L \end{bmatrix} \quad (10)$$

where $\epsilon_1 \dots \epsilon_J$ are the cost-based Domar weights for the regular industries, and ϵ_K and ϵ_L are the cost-based Domar weights for capital and labor. What BF prove is that in this setting ϵ_K and ϵ_L are the elasticity of aggregate output with respect to capital and labor, respectively, to a first-order approximation. As this calculation holds for any period t , the elasticities are more properly denoted ϵ_{Kt} and ϵ_{Lt} .

It is worth considering the intuition behind this result. In equation (9) the term $(I - \Lambda)^{-1}$ is the Leontief inverse. Let ℓ_{ij} denote the typical element of the Leontief inverse. ℓ_{ij} captures the elasticity of output in industry i with respect to a productivity shock in industry j , accounting for all the input-output linkages joining them.⁵

In the case of capital and labor, these “industries” have no final use and only serve as suppliers of an input to other industries. A productivity shock to these factor “industries” is nothing more than an increase in their supply. Hence the values of ℓ_{iK} and ℓ_{iL} show us the elasticity of output in industry i with respect to the supply of capital or labor, respectively. This elasticity incorporates not just the direct effect of more capital or labor on output in industry i (which is captured by the cost shares λ_{iK} and λ_{iL}), but also incorporates the indirect effect of increased factor inputs on the output of suppliers to industry i , on the output of suppliers to the suppliers of industry i , and so on.

BF show that that ℓ_{iK} and ℓ_{iL} measure the elasticity of real output (as opposed to revenue) with respect to capital and labor so long as the original entries in Λ are *cost* shares and not value-added shares. This is the same insight from Hall (1988, 1990), Basu and Fernald (2002), and Fernald and Neiman (2011) regarding the use of cost shares to measure elasticities, but applied in a disaggregated manner to an economy with input-output linkages between industries.

The aggregate effect of an increase in capital or labor depends on the individual terms ℓ_{iK} and ℓ_{iL} weighted by the value-added share of each industry. From the matrix multiplication in equation (9) the aggregate elasticity with respect to each input can be expressed as

$$\begin{aligned} \epsilon_{Kt} &= \sum_{i \in J} va_{it} \ell_{iKt} \\ \epsilon_{Lt} &= \sum_{i \in J} va_{it} \ell_{iLt}. \end{aligned} \quad (11)$$

This breakdown of the elasticities will be useful later in decomposing the change in the elasticities over time. Overall, the BF structure makes it possible to calculate the elasticity of aggregate

⁵Carvalho and Tahbaz-Salehi (2018) provide a very nice introduction to production networks and the interpretation of the Leontief inverse.

output with respect to aggregate capital or labor without an aggregate production function, with rich input/output structures, and with arbitrary markups at the industry level.

3 National accounts data

The calculation of ϵ_{Kt} and ϵ_{Lt} is straightforward in theory, but not in practice. The well-known issue is the disconnect between what is reported in the national accounts (e.g. gross operating surplus) and what is necessary for the calculation (e.g. the cost of capital). This disconnect is why in the following section I will build bounds for estimates of ϵ_{Kt} (and ϵ_{Lt}) based on different ways of reconciling the national accounts with the needs of the calculation.

Prior to those main results I explain the three main U.S. data sources, set notation regarding the national accounts, and explain the process I used to merge together various sources to create industry-level data that spans 1948-2018 for the United States.

The first source of data are input/output tables. These are the Bureau of Economic Analysis (BEA) Use Tables, before redefinitions, at producer value (U.S Bureau of Economic Analysis, 2020*b*). These yearly use tables provide the information on the costs of intermediates purchased by industry i from industry j in year t , or $COST_{ijt}$ in the terminology from the prior section. They also include data on the value-added of each industry i , which I denote as $VALU_{it}^{IO}$, and where the IO superscript refers to the source of this data.

Industries in the input/output tables are all classified according to the NAICS 2012 system, but due to data limitations the BEA provides the tables at different levels of aggregation depending on the year. For 1947-62 they report 46 industries, for 1963-1996 65 industries, and for 1997-2018 71 industries.

The second source of data are industry-level components of value-added from the BEA national income and product accounts (U.S Bureau of Economic Analysis, 2020*e,a*). Specifically, I collect measures of value-added, $VALU_{jt}^{NIPA}$, labor compensation, $COMP_{jt}^{NIPA}$, proprietors income, $PROP_{jt}^{NIPA}$, and taxes and subsidies, TAX_{jt}^{NIPA} for each industry j in year t . The superscript $NIPA$ refers to the source of this data. Last, note that this data is sub-scripted by j (not i) to indicate that the industrial classification of this NIPA data may be different than the industrial classification in the IO table.

To impute labor compensation data (for example) from NIPA to the IO table, I use the following equation,

$$COMP_{it}^{IO} = VALU_{it}^{IO} \times \frac{COMP_{jt}^{NIPA}}{VALU_{jt}^{NIPA}}. \quad (12)$$

To implement this I need to match each industry i in the IO table to an appropriate industry j in the NIPA data. Given that match, I use the ratio of compensation to value-added in the NIPA

industry to impute the size of compensation in the IO industry. A similar expression is used for both proprietors income and taxes and subsidies.

Table 9 documents the classification schemes used by different sources (IO, NIPA) for different years. For 1997-2018 the NIPA data (U.S Bureau of Economic Analysis, 2020*e*) are classified according to the NAICS 2012 system, and thus it is possible to match an industry i in the IO table to the exact same industry j in the NIPA data. Further, the value-added reported in the two sources are identical for these years, and hence the expression above devolves to $COMP_{it}^{IO} = COMP_{jt}^{NIPA}$ (and similar for the other components of value-added).

For 1948-1996, however, NIPA data are not reported according to the NAICS 2012 classification. Industry-level data from the *Historical Industry Accounts Data* (U.S Bureau of Economic Analysis, 2020*a*) is classified by either SIC 1972 or SIC 1987 industries, depending on the year. I continue to use the equation above to find compensation (and proprietors income and taxes) in industry i of the IO table, but this now requires making assumptions about which industry j in the NIPA data provides an appropriate match to industry i in the IO data in year t .

I rely on crosswalks between SIC 1972, SIC 1987, and NAICS 2012 classifications, and my own judgment to make these matches. A straightforward example from 1960 would be using “Transportation by air” (SIC 1972 code 45) as industry j from NIPA to match to “Air transportation” (NAICS 2012 code 481) as industry i in the IO table.

The matching for a given year is not always one-for-one, and there are NIPA industries j (e.g. SIC 1972 code 73 “Business services”) whose ratios are used for multiple industries i in the I/O table (e.g. NAICS code 561 “Administration and support services”, NAICS code 55 “Management of companies”, etc.). There are also situations where I have aggregated the data from NIPA industries (e.g. SIC 1972 codes 63 “Insurance carriers” and 64 “Insurance agents, brokers, and service”) and then matched this aggregate to an industry i in the IO table (e.g. NAICS code 524 “Insurance carriers and related activities”).

Full details of the matching are available in the Appendix. I have experimented with a variety of different reasonable choices for matching, and have not found any that change the results of the paper in an appreciable way.⁶ The Appendix also shows that the estimates of ϵ_{Kt} and ϵ_{Lt} do not have any distinct jumps in years when the source data change classification schemes (e.g. from 1996 to 1997).

The final data source I use are the Fixed Asset Accounts Tables of the BEA (U.S Bureau of Economic Analysis, 2020*d,f*). These provide information on the size of the capital stock of type k in industry j at years t , K_{jkt} , the amount of depreciation, $DEPR_{jkt}$, and investment spending, INV_{jkt} . The three types of capital k reported are structures, equipment, and intellectual property. This fixed asset data is reported according to the NAICS 2012 classification, and so can be matched

⁶Code and instructions are available on my website to the reader who wishes to experiment with different matching assumptions between the NIPA and IO table sources.

directly to the IO table industries.⁷

Once combined the dataset contains yearly information from 1948-2018, at the NAICS 2012 classification level, of industry-level information on intermediate costs, value-added, labor compensation, proprietors income, taxes and subsidies, depreciation of capital (by type), investment in capital (by type), and the stock of capital (by type).

4 Bounding the aggregate elasticities

As mentioned above, there is a disconnect in the presentation of data in the national accounts and the requirements of the calculation of ϵ_{Kt} and ϵ_{Lt} from equation (9). In short, this is an issue of how to split gross operating surplus into labor costs (part of proprietors income), capital costs, and economic profits. As there is no clear correct answer for how to do this, my approach is to construct several estimates of ϵ_{Kt} and ϵ_{Lt} based on different assumptions.

Two of these assumptions will form what I consider to be natural bounds on the size of ϵ_{Kt} and ϵ_{Lt} . Assuming that there are zero economic profits in the economy will maximize the implied cost of capital imputed from the national accounts data, and hence lead to an upper bound for ϵ_{Kt} (and a lower bound for ϵ_{Lt}). On the other hand, we know that industries experienced the depreciation of existing capital and their capital costs are at least this large. Using depreciation costs to impute the cost of capital will therefore give a lower bound for ϵ_{Kt} (and an upper bound for ϵ_{Lt}).

Neither of these bounds are necessarily unbreakable. In the presence of *negative* economic profits the implied cost of capital could be even higher than supposed with the zero-profit assumption. Or if the user cost of capital were to become negative (perhaps due to high expected inflation) it is possible capital costs could be lower than what is captured by depreciation. But alternative assumptions regarding capital costs, such as a user cost calculation, do tend to fall within the bounds I establish.

From the point forward I focus almost exclusively on discussing the capital elasticity, ϵ_{Kt} . The labor elasticity ϵ_{Lt} is simply one minus the capital elasticity, so the implications for the labor elasticity are straightforward to infer.

4.1 Labor costs, proprietors income, and taxes

The first step is to deal with the measurement of labor costs, $COST_{iL}$. I follow Gomme and Rupert (2004) and assign a share of proprietors income and production taxes to labor equal to the ratio of reported compensation to non-proprietors value-added:

⁷There are minor discrepancies between the NAICS classifications in the Fixed Asset Accounts and the IO tables. These are straightforward to manage in that the fundamental classification system is the same. Details are in the Appendix.

$$COST_{iLt} = COMP_{it} + (PROP_{it} + TAX_{it}) \left(\frac{COMP_{it}}{VALU_{it} - TAX_{it} - PROP_{it}} \right), \quad (13)$$

where $COMP_{it}$ is reported labor compensation in industry i at time t , $PROP_{it}$ is proprietors income, $VALU_{it}$ is value-added, and TAX_{it} is taxes and subsidies. Gomme and Rupert (2004) argue that this provides a more accurate representation of the labor component of proprietors income than using the number of self-employed workers and a measure of average wages, as proprietors are likely to be high productivity (and hence high wage). This also apportions production taxes to labor in the same manner.

In the Appendix I show variations on this assumption where I either assign all proprietors income as labor costs (i.e. $COST_{iLt} = COMP_{it} + PROP_{it}$) or all proprietors income as capital cost (i.e. $COST_{iLt} = COMP_{it}$). In the former case, the results are quite similar to the baseline using the Gomme and Rupert approximation. In the latter case the implied costs of capital are higher, implying higher measures of ϵ_{Kt} (and lower estimates of ϵ_{Lt}). However, this seems an unlikely case, given the widespread use of adjustments to proprietors income that assign much of it as labor income. Hence I use the Gomme and Rupert adjustment in (13) as my baseline throughout the paper.

4.2 Boundaries for capital costs

With the labor cost determined, it now remains to impute capital costs, $COST_{iKt}$, for each industry in each year.

4.2.1 No profit assumption

The first assumption is that there are zero economic profits. This means that gross operating surplus minus any adjustments for proprietors income and taxes represents a payment to capital. To be specific, for this assumption I set

$$COST_{iKt}^{NoProf} = VALU_{it} - COST_{iLt}. \quad (14)$$

Given this cost of capital for an industry, I can then calculate the capital cost share, λ_{iKt}^{NoProf} . In addition, I can calculate the cost shares of various intermediates, λ_{ijt} , and labor, λ_{iLt} , once those capital costs are set.

With the cost shares in hand, it is straightforward to calculate ϵ_{Kt}^{NoProf} given the formula in (9). As the aggregate capital elasticity is effectively a weighted average of the industry-specific capital cost shares, by using the no-profit assumption ϵ_{Kt}^{NoProf} I have an upper bound on the aggregate capital elasticity.⁸ The same calculation provides an estimate of ϵ_{Lt}^{NoProf} , but represents a lower

⁸Theoretically, this calculation is overkill. With zero profits Baqaee and Farhi (2019, 2020) show that the aggregate

bound for that elasticity.

Figure 1 plots the estimated values of ϵ_{Kt}^{NoProf} over time for the United States as the heavy black dashed line. In Figure 1 one can see that the no-profit capital elasticity begins at just over 0.32 in 1948 and rises with mild fluctuations to a value of 0.39 by 2018. This no-profit upper bound for the capital elasticity tracks the value of 1/3 (with a mild dip in the 1970s) from 1948-1995. The the value of 1/3 would only be appropriate in those years if one believed there were zero economic profits in the economy. After 1995, there is shift up in the no-profit upper bound to about 0.38-0.39, making the value of 1/3 appear more plausible. Table 2 provides summary statistics of the estimated capital elasticities

4.2.2 Depreciation costs only

The second estimates of the aggregate elasticities are made using depreciation to impute the cost of capital. Depreciation by itself misses costs associated with the on-going financing of the capital stock by firms, but has the advantage of being reported on an industry-by-industry basis. We know that industries face *at least* this depreciation cost. From that perspective, using depreciation provides a plausible lower bound for capital costs.

In terms of the structure outlined above, I measure capital costs in industry i as follows

$$COST_{iKt}^{Depr} = DEPR_{it}. \quad (15)$$

As this point the logic is identical to the prior sub-section. These costs allow me to calculate cost shares for each industry, and those cost shares are used in equation (9) to calculate the aggregate capital elasticity, ϵ_{Kt}^{Depr} , and the aggregate labor elasticity, ϵ_{Lt}^{Depr} .

In Figure (1) the capital elasticity ϵ_{Kt}^{Depr} is plotted from 1948-2018 as the solid black line. As expected, this series lies everywhere below the no-profit estimates. The estimated elasticity begins at 0.16 in 1948 and finishes at 0.26 in 2018, for an increase of 0.11 that is very similar to the increase in the no-profit upper bound.

From the figure it is also apparent that the gap between the no-profit upper bound and the depreciation-based lower bound remains roughly constant. That gap averages 0.13, and is as low as 0.091 (1980) and as high as 0.174 (1951). In the Figure that range is shaded in light gray to indicate the plausible values for ϵ_{Kt} in any given year. Note that this range is not a confidence interval and nothing about it implies that the actual capital elasticity lies in the middle of the range.

capital elasticity is equal to capital's share of GDP.

4.3 Alternative capital cost estimates

The prior sub-section showed the boundaries over time, and here I present two more estimates of the capital elasticity that do not make sense as bounds, but provide information on the actual path of the elasticity and whether those bounds are sensible.

4.3.1 Investment costs

Here I use observed investment spending by each industry as the measure of capital costs. One way this imputation may be sensible is to consider an economy following a “Golden Rule”, where all capital income is used to purchase capital goods. As capital income to one person represents a capital cost to another, investment spending would measure capital costs.

An additional advantage of the investment cost assumption is that this data is measured directly from industry level spending. Depreciation costs by themselves are estimated by the BEA based on depreciation schedules that may not accurately reflect true industry experience. Arguably the use of observed investment spending to measure capital costs may be the choice with the fewest assumptions built in.

In this case the cost of capital is measured as follows,

$$COST_{iKt}^{Inv} = INV_{it}. \quad (16)$$

Once again, the logic at this point is standard. Using these costs I obtain cost shares for capital, labor, and intermediates, and using (9) I can calculate ϵ_{Kt}^{Inv} and ϵ_{Lt}^{Inv} .

Figure (2) plots the estimated capital elasticity from 1948-2018 with the line marked with x’s, as well as the original bounds. This estimate begins at 0.22 in 1948 and runs to 0.295 in 2018, demonstrating a less dramatic increase than either the depreciation or no-profit bounds. Based on investment costs, the capital elasticity ends up at the lower depreciation-based bound around the time of the Great Recession in 2009 and remains close to that lower bound until 2018. This is consistent with investment spending by industry that acts solely to replace depreciating capital in that period.

Of note, the investment cost elasticity estimate remains everywhere inside the bounds set by the depreciation-only and no-profit estimates. The relatively small increase over time in the investment cost elasticity estimates reminds one that the true capital elasticity may well move between the bounds over time and does not necessarily increase just because the bounds do. In addition, the capital elasticity based on investment costs is everywhere below the value of 1/3.

4.3.2 User cost of capital

As a last alternative, I turn to the standard user cost of capital calculation of Hall and Jorgenson (1967). This has been used extensively to estimate the cost of capital, including in recent work

on labor and capital's share of aggregate GDP (Barkai, 2017), with the downside of needing to make several assumptions about the financing costs facing industries and expectations of inflation of capital goods.

The calculation of the overall cost of capital for an industry i is more complex than the prior assumptions. First, I allow for three types of capital goods - structures, equipment, and intellectual property - which is available from the BEA capital stock data (U.S Bureau of Economic Analysis, 2020*d,f*). Each industry i has a stock of capital of each type j at time t , K_{ijt} . Each industry i also faces a rental rate for capital of type j at time t , R_{ijt} , and these rental rates are allowed to vary by industry.

Overall, the cost of capital to industry i at time t is

$$COST_{iKt}^{User} = \sum_{j \in \{st, eq, ip\}} K_{ijt} R_{ijt}. \quad (17)$$

The rental rate for each type of capital in a given industry is given by

$$R_{ijt} = (Int_{it} - E[\pi_{ijt}] + \delta_{ijt}) \frac{1 - z_{jt}\tau_t}{1 - \tau_t} \quad (18)$$

where Int_{it} is the nominal interest cost of financing facing industry i at time t , explained below. $E[\pi_{ijt}]$ is the expected inflation in the price of capital type j for industry i at time t , and δ_{ijt} is the cost of depreciation. The term z_{jt} is the depreciation allowance for taxation of capital type j at time t , and τ_t is the effective corporate tax rate at time t .

In this baseline, the expected inflation rate $E[\pi_{ijt}]$ for a given capital type j in industry i at time t is just observed inflation rate in t , based on the price indices by capital type in the BEA fixed asset accounts (U.S Bureau of Economic Analysis, 2020*d,f*). In the Appendix I show that there are small changes to the results if I instead proxy expected inflation using forward-looking or backward-looking inflation over different spans (1-year, 3-year, 5-year).

The nominal interest rate facing industry i at time t is calculated as a weighted average of market interest rates for different types of financing (e.g. corporate bonds, equity, mortgages), denoted by Int_{mt} , and the weights vary by industry. Formally,

$$Int_{it} = \sum_m s_{imt} Int_{mt} \quad (19)$$

where s_{imt} is the share of financing of type m in industry j , and Int_{mt} is the observed interest rate on that type of financing.

While this is industry-specific, it primarily differentiates between government, housing, and the private sector. Full details are available in the Appendix on sources for the shares and rates used. A brief summary is that the federal government industry is assumed to be financed using 10-year Treasury bonds and housing is assumed to be financed using 30-year mortgages. Private industries

are financed using a combination of corporate AAA bonds, corporate Baa bonds, short-term loans, and equity, with the shares (s_{imt}) determined from industry-level balance sheets provided by the integrated macroeconomic accounts (U.S Bureau of Economic Analysis, 2020c).

Given the values of Int_{it} I am able to calculate the rental rate of capital facing each industry i for each capital type j , R_{ijt} , and then the overall cost of capital for industry i at time t , $COST_{iKt}^{User}$. With those costs of capital, I can then proceed in the same manner as before, and calculate the elasticities ϵ_K^{User} and ϵ_L^{User} using equation (9).

In Figure (2) the series of ϵ_K^{User} is plotted from 1948-2018 marked by o's. This is far more variable over time than the bounds set by depreciation and no-profit assumptions, as well as more variable than the estimates based on investment costs. However, the user cost of capital estimates stay for the most part inside the bounds set by depreciation and no-profit.

There are notable exceptions. In 8 years (1950, 1973, 1974, 1977, 1978, 2004, 2005, and 2013) the user cost of capital is below the bound set by the depreciation cost estimates. The observations in the 1970s are due to very high inflation of all capital goods, which in the user cost calculation results in very low rental rates and hence a low cost to capital. The 2004 and 2005 observations are due to very high inflation in structures. 1950 and 2013 appear to be a combination of slightly higher inflation in structures and low financing rates. Nevertheless, these deviations below the bound of depreciation cost estimates are not large and appear to be short-lived.

On the other end, there is a continuous stretch from 1981 through 1992 where the user cost estimates are above the upper bound set by the no-profit estimates. These are due to the relatively high nominal rates on financing during this period and the lower values observed for inflation on all capital goods. These deviations are not large after 1984. If the user cost estimates of the elasticities are correct, then they would imply negative economic profits during this period. Alternatively, the user cost calculations may not be accurately representing the cost of capital in this period.

Regardless, in 51 of the 71 years reported the estimate of ϵ_K^{User} falls between the bounds denoted by the no-profit and depreciation costs estimates. Over time, the trend of the user cost estimates appears to track the trend of the investment cost estimates, and both imply an elasticity that is below 1/3 for much of this time period.

5 Trends and relationships to aggregate values

None of the four estimated series of ϵ_{Kt} in Figure 2 are “right”. Without direct measurement of the cost of capital faced by industries, and better information on the split of proprietors income, any estimate of the aggregate elasticities is necessarily based on some assumptions. However, the combination of the various estimates can be used as guidance regarding plausible values of ϵ_{Kt} (and ϵ_{Lt}) over time. In addition, these estimates can be compared to “naive” estimates based on economy-wide ratios to see how relevant the input/output relationships embedded in equation (2)

may be.

5.1 Summary and trends

Table 2 summarizes the values of ϵ_{Kt} in Panel A (“Baseline”) for the four different assumptions regarding capital costs from the prior section. The mean, median, minimum, and maximum value from 1948-2018 of the capital elasticity are displayed in the first four columns. As can be seen, under the no-profit assumption the mean and median values of ϵ_{Kt} are highest, and it has the highest minimum value as well. In comparison, the depreciation assumption has the lowest mean and median, and the lowest maximum value of all the assumptions used.

Based on the median values, this implies that ϵ_{Kt} lies in a range of 0.220-0.334, and that the typical assumption of ϵ_{Kt} being equal to one-third is likely too big unless one is focused only on the period from around 2000-2018.

There is a suggestion in Figure 2 that the range of plausible capital elasticities may have risen over time. The final three columns of Table 2 provide more tangible estimates of that rise. For each assumption on capital costs, I ran the following simple OLS regression

$$\epsilon_{Kt} = \beta_0 + \beta_1 t + \nu, \tag{20}$$

where t is the year and ν is a noise term. The estimate $\hat{\beta}_1$ was used to construct the fitted increase in ϵ_{Kt} over time for that assumption, $\Delta\hat{\epsilon}_{48-18} = 70 \times \hat{\beta}_1$. This fitted change is reported in column (5), the value of $\hat{\beta}_1$ in column (6), and the R-squared of the regression in column (7).

One can see that for all four assumptions regarding capital costs in Panel A of Table 2, the fitted change was positive. For the no-profit assumption, the fitted value of the capital elasticity rose by 0.069, while for the depreciation cost assumption it rose by 0.106. The investment cost assumption had the smallest trend increase, or only 0.055.

None of this constitutes proof that the aggregate capital elasticity rose over time. It is quite plausible that the capital elasticity stayed constant over time but the noisy nature of the data has created an impression of trends. But if we accept the increase in the bounds as evidence of an upward trend in ϵ_{Kt} , note that this elasticity is almost certainly below 1/3 from 1948-1995, and below 0.4 from 1995-2018. It is also plausible that the capital elasticity was around 1/4 the entire time, given the evidence from the investment cost assumption.

5.2 Comparison to aggregate ratios

The estimates of the aggregate elasticity are built using industry-level data with input-output relationships, but it is informative to compare that elasticity to aggregated data on capital costs that ignores the industry input/output structure. Such aggregate data is what has been typically

used to estimate these elasticities in the past, and has the advantage of being much more readily available.

Using the notation developed above, define aggregate capital costs as a share of all factor costs (labor plus capital), s_{Kt}^{Cost} , as follows,

$$s_{Kt}^{Cost} = \frac{\sum_{j=1}^J COST_{jKt}}{\sum_{j=1}^J COST_{jKt} + COST_{jLt}}. \quad (21)$$

A second aggregate ratio to consider is aggregate capital costs as a share of value-added, s_{Kt}^{VA} , defined as

$$s_{Kt}^{VA} = \frac{\sum_{j=1}^J COST_{jKt}}{\sum_{j=1}^J va_{jt}}. \quad (22)$$

Note that both of these aggregate ratios still depend on the choice of assumption regarding capital costs.

Table 3 provides summary statistics for both s_{Kt}^{Cost} and s_{Kt}^{VA} in Panel A. Under the no-profit assumption in the first row, columns (1)-(4) the mean value of s_{Kt}^{Cost} is 0.337, and ranges from a minimum of 0.291 to a maximum of 0.391 between 1948 and 2018. In columns (5)-(8) the summary statistics for s_{Kt}^{VA} are identical for the no-profit assumption, by construction. With zero economic profits the denominators of the two ratios are identical.

Moreover, note that these summary statistics are identical to the estimates of ϵ_{Kt} under the no-profit assumption in Panel A of Table 2. This is again by construction. As Baqaee and Farhi (2019, 2020) derive, with zero economic profits the elasticity is equal to capital's share of value-added, even in the presence of input/output relationships.

Reading down the rows in Panel A of Table 3, however, there are distinctions between s_{Kt}^{Cost} and s_{Kt}^{VA} in the different scenarios, and the summary statistics of these ratios do not match the summary statistics of ϵ_{Kt} from Table 2. The differences in the aggregate ratios arise because of the presence of economic profits, which is why the value-added ratio s_{Kt}^{VA} is everywhere less than or equal to the cost ratio s_{Kt}^{Cost} . The difference of the cost ratio from the elasticity arises because of those profits and the input/output nature of the calculations.

That said, it turns out that the aggregate cost ratios are a decent approximation of the capital elasticity under different capital cost assumptions. Figure 3 plots the estimated values of ϵ_{Kt} for all years under three assumptions regarding capital costs against the corresponding ratio s_{Kt}^{Cost} for that assumption in the same year. The no-profit assumption is excluded because the elasticity and ratio are identical there by construction.

What can be seen is that the elasticity estimates lie generally above the corresponding ratios, with the 45-degree line denoted by dashes. The differences are distinct, but not very large in practice. The average difference between elasticity and ratio ($\epsilon_{Kt} - s_{Kt}^{Cost}$) using the investment cost

assumption is 0.014, 0.23 using the user cost assumption, and 0.034 using the depreciation cost assumption.

These differences reflect the influence of input/output relationships in raising the implied elasticity with respect to capital over and above what the simple ratio would suggest. In other words, industries with large capital elasticities tend to be supplies of other industries, which increase the aggregate elasticity of the economy with respect to capital. However, the size of the differences suggests that one could derive a reasonable approximation from the ratio alone, which may be useful in the absence of an input/output table. Knowing the downward bias of the ratio relative to the elasticity would be informative in such a situation.

5.3 Accounting for change

The bounds on ϵ_{Kt} rose over time. By themselves, these changes in the bounds do not necessarily imply that the actual values of ϵ_{Kt} and ϵ_{Lt} changed. But it seems worth exploring what drove the changes in the bounds, as they imply shifts in the aggregate elasticities were plausible.

To account for the change in bounds, return to equation (12) and the expression showing how ϵ_{Kt} is a weighted sum of entries from the Leontief inverse,

$$\epsilon_{Kt} = \sum_{j=1}^J va_{jt} \ell_{jKt}.$$

To track the changes in ϵ_{Kt} over time I perform an Olley and Pakes (1996) decomposition on this summation, yielding

$$\epsilon_{Kt} = \bar{\ell}_{Kt} + \sum_{j=1}^J (va_{jt} - \bar{va}_t)(\ell_{jKt} - \bar{\ell}_{Kt}), \quad (23)$$

where $\bar{\ell}_{Kt}$ is the unweighted mean of the Leontief elements for capital. This mean industry-level elasticity shows how sensitive industries are to capital, ignoring their share of GDP. Tracking this over time will indicate whether industries in general were becoming more or less sensitive to the use of capital.

The summation term above is the ‘‘covariance’’ of value-added shares and Leontief elements. When positive, it indicates that industries that are more sensitive to capital (e.g. have ℓ_{iKt} above average) also tend to be large in value-added terms. When negative, it indicates that industries sensitive to capital are relatively small. Tracking this covariance term over time will show whether capital-sensitive industries were becoming larger or smaller.

Figure 4 plots the values of ϵ_{Kt} and $\bar{\ell}_{Kt}$ for both the upper and lower bounds of the elasticity, which are determined by the no-profit and depreciation cost assumptions on capital costs. The covariance is not plotted separately but can be inferred from examining the Figure, as it is the gap

between plotted series.

For the no-profit upper bound, it is apparent that the mean industry-level elasticity, $\bar{\ell}_{Kt}$, drove the drift up over time. The gap between ϵ_{Kt} and $\bar{\ell}_{Kt}$ is accounted for by the covariance term in (23), which is positive but small in absolute size throughout. The upper bound on the aggregate elasticity rose over time because, on average, most industries were getting more sensitive to capital.

This story is repeated with the depreciation cost lower bound. Again the mean industry-level elasticity, $\bar{\ell}_{Kt}$, lies everywhere below the aggregate elasticity, ϵ_{Kt} , which implies again a small positive covariance term. The drift upward is due to higher mean capital elasticities at the industry level, and not due to changes in the covariance between value-added shares and industry elasticities.

6 The composition of capital and output

Up to this point I have been working with data that covers all industries, including those for which value-added and capital costs may be particularly hard to measure correctly (e.g. government, owner-occupied housing). The baseline also takes as given the capitalization of intellectual property in the national accounts, which is a recent adjustment that impacts value-added and capital stocks within industries.

6.1 Private business sector

To see the influence of the inclusion of government (both general government and government enterprises at federal and state/local levels) and owner-occupied housing on the aggregate elasticities, I remove them both from the calculation of ϵ_{Kt} and ϵ_{Lt} . In practice, this means deleting their rows and columns from the IO matrix Λ as well as their entries from the vector of value-added shares in Γ .⁹ This makes the coverage of the calculation equivalent to the “Private business sector” coverage that the BLS uses.

Figure 5 plots the estimated value of the capital elasticity bounds (the no-profit and depreciation cost assumptions) for the private business sector as dark black lines, and the plausible range for the capital elasticity is shaded in dark gray. For comparison, the original bounds using all industries are plotted using the dashed lines, and that range is shaded in light gray.

As can be seen, the range of the capital elasticity for the private business sector lies everywhere lower than the range for the aggregate economy. In general, both bounds are shifted down by approximately 0.05. Notably, the capital elasticity for the private business sector lies definitively

⁹In practice there are several industries that are deleted, depending on the year. NAICS includes entries for federal general government (defense), federal general government (non-defense), federal government enterprises, state and local general government, and state and local government enterprises. Prior to 1997, the federal general government categories are combined into a single industry. With respect to housing, both housing and other real estate are excluded. Prior to 1997, those two industries were aggregated into a single real estate industry.

under 1/3 throughout the period, and only during the years 2010-2018 does the upper bound reach that value. The naive value is too high if one is considering just private business sector activity.

In Table 2, Panel B, summary statistics are reported for the private business sector alone. Comparing Panel B to Panel A, the downward shift of about 0.05 in the aggregate capital elasticity shows up regardless of how capital costs are calculated. This has the implication that the aggregate labor elasticity is estimated to be higher by about 0.05 in the private business sector.

This shift in elasticities is driven almost entirely by the exclusion of housing. In the Appendix I show results that exclude housing and government separately, and one can see that it is only after excluding housing that this significant drop in ϵ_{Kt} takes place.

Mechanically, the lower elasticity in the private business sector comes from the fact that the housing industry has a very high capital cost share under any set of assumptions. The average capital cost share is 0.94 under the no-profit assumption, 0.89 under the investment cost assumption, 0.83 under the user cost assumption, and 0.80 under the depreciation-only assumption. Full statistics on these shares are available in the Appendix. The estimates of ϵ_{Kt} are weighted averages of cost shares across different industries (with the weights depending on input-output linkages), so the exclusion of owner-occupied housing lowers ϵ_{Kt} for the private business sector.

If we restrict ourselves to the private business sector of the economy then the likely size of ϵ_{Kt} is well below one-third, and even below 0.30. Nevertheless from Table 2, Panel B, columns (5)-(7) one can see that the elasticity under all assumptions did rise over time, and in amounts very similar to the rise in the elasticity when the entire economy is considered in Panel A. The private business sector has a similar trajectory of ϵ_{Kt} over time, but is shifted down compared to the overall economy.

6.2 De-capitalizing intellectual property

A related issue concerns the inclusion of intellectual property as a capital stock in the national accounts. As Koh, Santaeuilàlia-Llopis and Zheng (2020) showed, the revision to the national accounts to capitalize intellectual property, begun by the BEA with their 11th revision in 1999, can explain essentially all of the reported decline in labor's share of GDP. While I am concerned here with the elasticity of GDP with respect to capital (and labor), the same features of the national accounts that Koh, Santaeuilàlia-Llopis and Zheng (2020) identified may be relevant here.

In particular, in capitalizing intellectual property (as opposed to treating it as an expense), the BEA revised up the value-added of each industry by an amount equal to the sum of own-account and purchased intellectual property. This also revised gross operating surplus by incorporating own-account intellectual property, and revised total depreciation to include that of intellectual property. Following Koh, Santaeuilàlia-Llopis and Zheng (2020), I reverse these modifications to strip out the capitalization of intellectual property and then estimate ϵ_{Kt} . Details of the calculations required are in the Appendix.

De-capitalizing IP changes the estimated aggregate elasticities. Figure 6 plots in the dark lines the upper (no-profit) and lower (depreciation-only) bounds for the elasticity ϵ_{Kt} when intellectual property is de-capitalized from the national accounts. For comparison purposes, the dashed lines plot the upper (no-profit) and lower (depreciation-only) bounds under the baseline situation where IP is considered a capital good.

As can be seen, there is a distinct shift down in the range of plausible ϵ_{Kt} values when IP is de-capitalized. The upper bound is well below $1/3$ throughout most of the time period, and only rises above it in 2005-2018, and even then the difference is small. There is a similar story for the lower bound, which starts similar to the baseline in 1948, but remains much lower through 2018. Figure 6 indicates that an important part of the apparent rise in ϵ_{Kt} over time was the capitalization of IP.

In Table 2, Panel C, I show the summary statistics when IP is de-capitalized. The mean and median values are lower under all assumptions compared to the baseline, by about 0.02-0.03. Perhaps more interesting is columns (5)-(7), which show that the implied rise in ϵ_{Kt} over time was more muted when IP is de-capitalized, but it does not disappear completely.

The mitigation of the upward trend is consistent with the findings of Koh, Santaeulàlia-Llopis and Zheng (2020) on the mitigation of the *downward* trend in labor’s share of GDP. In both cases de-capitalizing IP leaves the size of labor compensation the same, but lowers the size of value-added in each industry. In my case, this implies that there is less value-added “left over” to be attributed to capital costs, capital cost shares are lower across industries, and hence the size of ϵ_{Kt} is lower.

Panel D of Table 2 shows summary statistics when IP is de-capitalized in the private business sector. The combined effect is to push the no-profit upper bound down lower, to an average of 0.258 for ϵ_{Kt} , with a maximum of 0.297, below $1/3$. The depreciation lower bound averages only 0.120, with a maximum of 0.165. With this narrow definition of economic output and excluding IP capital, it is plausible that the capital elasticity was below 0.3 on a regular basis.

7 Comparison with the OECD

Given the time series for the U.S., a natural question is whether the values of ϵ_{Kt} (and of ϵ_{Lt}) are of similar sizes in other countries. I use data from the OECD STAN database (Horvát and Webb, 2020) and input/output tables (OECD, 2020) to calculate values of ϵ_{Kt} for a set of 20 OECD countries in the period 2005-2015. The limited time frame compared to the baseline U.S. case is due to lack of full data for OECD countries prior to 2005. The OECD dataset does include the U.S., however, and so this will provide a separate point of comparison to the baseline estimates.

There are other limitations to the OECD data. First, there is no information regarding proprietors income, and hence there is no way to apply the Gomme and Rupert (2004) adjustment to recover total labor compensation. The OECD does report total compensation, total employees, and

the total number of self-employed workers. Using that information I create compensation measures for industry i in country m in year t as follows,

$$COST_{miLt} = COMP_{mit} + SELF_{mit} \left(\frac{COMP_{mit}}{EMPL_{mit}} \right). \quad (24)$$

$COMP_{mit}$ is reported labor compensation, $SELF_{mit}$ is the number of self-employed workers, and $EMPL_{mit}$ is the number of regular workers. This adjustment thus uses the average wage to value the labor compensation of self-employed workers. As Gomme and Rupert (2004) point out, this likely understates labor compensation to the extent that self-employed workers are owner-operators who likely bring more skill or human capital to the firm. This implies that the estimates of ϵ_{mLt} will likely be understated, and the estimates of ϵ_{mKt} overstated for the OECD countries.

The second limitation on the OECD data is a lack of information on the financial structure of liabilities by industry in different countries, preventing me from calculating a user cost of capital measure comparable to what I used for the U.S.. In the absence of this information, I only make comparisons using the no-profit, depreciation cost, and investment cost assumptions. For the U.S. I calculate ϵ_{mKt} using the OECD data and method for handling self-employment so that it is comparable to other OECD countries. I will also compare this U.S. calculation to the one done previously using BEA data.

Caveats aside, the process for calculation is again using equation (9). Table 4 provides the mean value of ϵ_{mKt} from 2005-2015 for each country, and the absolute change from 2005 to 2015 in each country, for the three available assumptions regarding capital costs. In addition, two rows at the end of the table show the mean over all OECD countries from 2005-2015, as well as the median across all OECD countries from 2005-2015. The final row of the table reports the values for the U.S. made using the baseline data from the BEA discussed in the prior sections of the paper, for comparison.

The values for the U.S. from the two sources deviate to some extent. The depreciation cost lower bound is 0.266 using the OECD, and 0.256 using the baseline BEA data, which are quite close. The no-profit upper bound is 0.431 using the OECD, and only 0.381 using the baseline BEA data. The difference in the upper bound is driven by the different treatment of proprietors income. This suggests that the OECD estimates for the upper bound may be skewed upwards by imputing self-employed labor income using the average wage, rather than the Gomme and Rupert (2004) adjustment for proprietors income. By understating labor costs this overstates capital costs in the no-profit scenario, leading to higher values for the capital elasticity.

Unfortunately there is no information in the OECD database for investment by industry in the U.S., so there is no way to make a comparison under this assumption about capital costs.

Comparing the U.S. OECD estimates to the rest of the OECD, the depreciation lower bounds are quite similar. The OECD mean lower bound is 0.280 in this period, and the median is 0.276.

For the no-profit upper bound, the OECD values are also similar to the U.S. value from the OECD: a mean of 0.441 for the OECD compared to 0.431 for the United States. Given the apparent upward bias in the no-profit upper bound noted for the U.S., it seems likely the upper bounds for all OECD countries are overstated to some extent.

That said, one can see variation across the OECD in the range of ϵ_{mKt} . The United Kingdom has a lower bound of only 0.221, and an upper bound of 0.359. At the other end of the scale, the Czech Republic has a lower bound capital elasticity of 0.322 and an upper bound of 0.503. But across the 20 OECD countries, the bounds on the values of ϵ_{mKt} are not disjointed, in the sense that every country's range of plausible values overlaps with every other country's.

One can see this in Figure 7, which shows the ranges of all 20 countries. The U.S. values are shown at the top, with o's representing the estimated elasticity in each year using the depreciation-only assumption (the lower bound), and the x's showing the estimated elasticity in each year under the non-profit assumption (the upper bound). The dashed vertical lines are the median lower and upper bound values for the U.S. between 2005-2015. Below the U.S. each country is shown in reverse alphabetical order, with the same interpretation of the symbols. While the ranges for some countries extend above 0.50, or as low as 0.20 for others, one can see that for the most part all the ranges overlap in the area between 0.30 and 0.40.

These estimates suggest some consistency across the OECD in the *range* of ϵ_{mKt} , but it should be noted that the true value of ϵ_{mKt} could differ across countries due to differences in market power or the true costs of capital. All this data suggests is that the plausible values are similar. The OECD data is calculated over the whole economy, including government and housing, and incorporates IP capital. Similar changes to those seen in the U.S. data when those industries or capital type are excluded seem likely. All that being said, the cost structures of industries appear to be similar to one another across countries in the OECD, such that the range of ϵ_{mKt} is also similar.

8 An application to growth accounting

Returning to the U.S. setting, the bounds I have found for the capital and labor elasticities are relevant to the calculation of total factor productivity (TFP) growth. In particular, common series on TFP growth assume that the labor elasticity can be estimated from labor's share of GDP, and that capital's elasticity is one minus the labor elasticity. These elasticities correspond to the "no-profit" scenario I use, and are only applicable if the economy has zero markups or market power. These series thus provide a bound on TFP over time, but may not reflect actual TFP growth. Here I show TFP growth over time when different assumptions about the elasticities are used.

The baseline calculation I am working with is as follows,

$$d \ln TFP_t^s = d \ln Y_t - \epsilon_{K_t}^s d \ln K_t - \epsilon_{L_t}^s d \ln L_t. \quad (25)$$

The difference in log TFP at time t , $d \ln TFP_t^s$, depends on the difference in log output, $d \ln Y_t$ minus the effects that are accounted for by capital growth, $d \ln K_t$, and growth in labor inputs, $d \ln L_t$. The data on output and inputs I take directly from the BLS. Growth in the labor input is made up of two parts, the growth rate of hours and the growth rate of labor quality, the latter of which is imputed by the BLS from the composition of the workforce and relative wages. Capital growth is measured as the growth in capital services, also imputed by the BLS.¹⁰

The superscript s in equation (25) refers to the assumption used to calculate the elasticities $\epsilon_{K_t}^s$ and $\epsilon_{L_t}^s$. I calculate TFP growth for four different values of s corresponding to series I described above in Section 4: no-profit, depreciation only, investment costs, and user costs. By construction, the series of TFP growth calculated using the values of elasticities under the no-profit assumption matches the standard BLS exactly (with minor rounding errors). The other assumptions s yield different series for $d \ln TFP_t^s$.

It is not obvious ex ante whether the growth rates of TFP will be higher or lower than the BLS baseline when I used different assumptions for the elasticities. Overall, the other assumptions give lower values for the capital elasticity, and higher values for the labor elasticity. Whether this leads to higher or lower estimates of TFP growth depends on the relative size of capital growth and labor growth. To the extent that capital growth is *higher* than labor growth, this will tend to lead to *higher* estimates of $d \ln TFP_t^s$ as the elasticities will reduce the implied role of input growth.

Figure 8 shows the results of the exercise. I've converted the growth rates of TFP into a level series with 1947 equal to 100, to aid in visualization. What becomes clear is that the no-profit assumption behind the BLS elasticities gives a *lower* bound to the level of TFP over time. Elasticities based on depreciation costs yield an *upper* bound to the TFP series, while the investment cost and user cost elasticities give series lying in between, consistent with the fact that those elasticities also tend to lie in between the bounds.

In terms of implication, it is likely that the standard BLS numbers are under-stating the size of TFP over time, and therefore the absolute growth rate of TFP. By 2018 under the BLS no-profit assumption TFP is 2.5 times higher than in 1947, but using the depreciation cost elasticities TFP is 3 times higher, 20% higher than the BLS baseline estimate. TFP growth under the BLS baseline average 1.29% per year, but could be as high as 1.58% per year given the depreciation only elasticities.

Table 5 shows the average annual growth rate of TFP, by decade, under different scenarios.

¹⁰To impute capital service the BLS allocates total capital costs across different types of capital. An implicit assumption in that imputation is that total capital costs are equal to all non-labor value-added, or that there are no profits. I take the BLS capital numbers as given, and only focus on the effect of changing the elasticities in equation (25).

Column (1) uses the BLS no-profit scenario, and shows the standard pattern of TFP growth over time (e.g. slowdown in the 2000s), and the overall average growth rate of 1.29% for 1948-2018. In columns (2)-(4) are the average annual growth rates calculated under the other scenarios for capital costs, and shows that these imply higher growth rates across all decades, but that the variation between decades is preserved. There is still a slowdown starting in the mid-1970s to the mid-1980s, followed by rapid growth in the 1990s, and then a slowdown in growth starting in the early 2000s. The magnitudes of these changes are different using different assumptions on the elasticities, but that does not overturn the general story regarding TFP growth.

9 Conclusion

The elasticities of GDP with respect to capital and labor are central parameters to almost any model of the economy. Values for these elasticities have traditionally been derived from factor share information, leading to the rule of thumb that the capital elasticity is equal to one-third and the labor elasticity two-thirds.

That rule of thumb requires several strong assumptions, including the existence of an aggregate production function and zero economic profits. In this paper I applied the theory of Baqaee and Farhi (2019, 2020) to the calculation of these elasticities, which eliminates those strong assumptions, and allows me to estimate the aggregate elasticities using industry-level data on costs of capital and labor.

Because of the standard problem of finding capital costs from national accounts data, I create bounds on the elasticities based on different assumptions. An upper bound for the capital elasticity is created by assuming there are zero economic profits, and a lower bound by assuming that depreciation is the only cost of capital. Those bounds indicate a value of the capital elasticity that was 0.18-0.33 from 1948-1995 in the U.S., and 0.21-0.39 from 1995-2018. If I limit the scope of the economy to the private business sector, or de-capitalize intellectual property from the national accounts, those bounds are shifted down by between 0.03-0.07 in each year. Comparing the U.S. to the OECD, there is substantial overlap in the estimates of the capital elasticity.

The results suggest that the rule of thumb “alpha equal one-third” is likely to overstate the size of the capital elasticity, at least for much of the time frame considered. This has consequences for things such as growth accounting. By using the bounds I calculate for the capital elasticity, I show that common BLS estimates of the TFP growth rate and level are likely understated.

While the overall results would suggest that the rule of thumb overstates the capital elasticity (and understates the labor elasticity), it is not wildly inaccurate. Going forward, papers that require an estimate of the capital and labor elasticities could use the boundaries I have calculated as part of robustness and sensitivity checks to confirm their results are not due to the specific elasticities chosen. Also valuable is that these aggregate elasticities that I have calculated are summary

measures for economies with arbitrary amounts of market power and input/output relationships (Baqae and Farhi, 2019, 2020). They can be applied in different models as a “reduced form” way of introducing the production side of the economy, without having to specify it in detail. More generally, studies relying on aggregate elasticities as part of calibration or imputations of productivity would be advised to consider the range of values estimated here to ensure they are not basing their findings on extreme values.

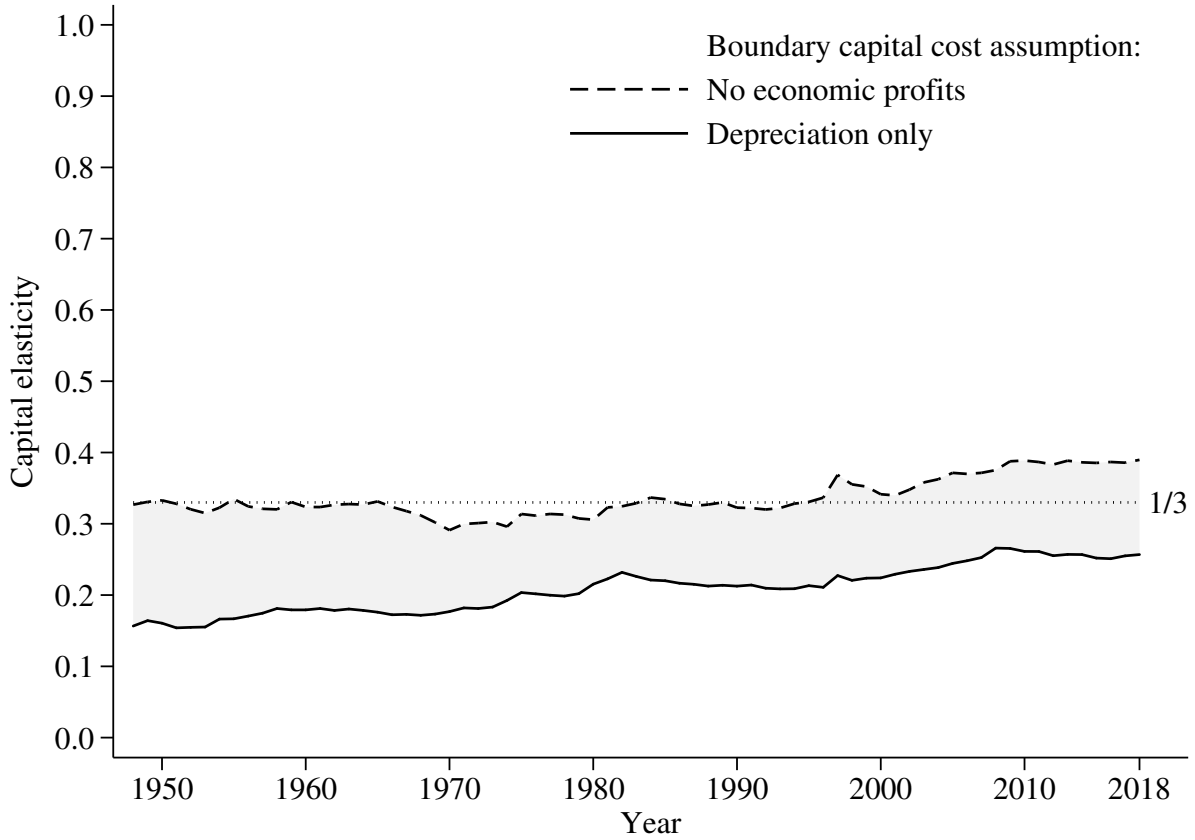
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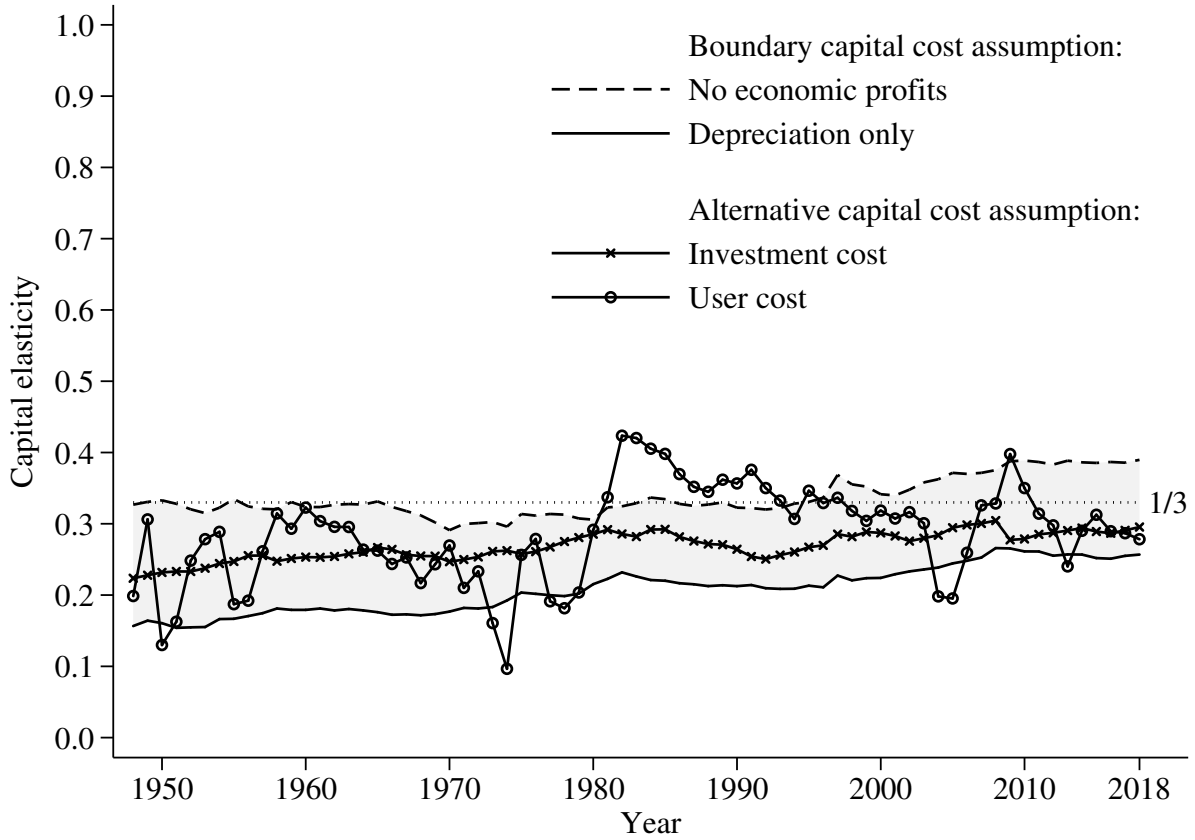
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Figure 1: Boundaries for the aggregate capital elasticity, ϵ_{Kt} , U.S. 1948-2018



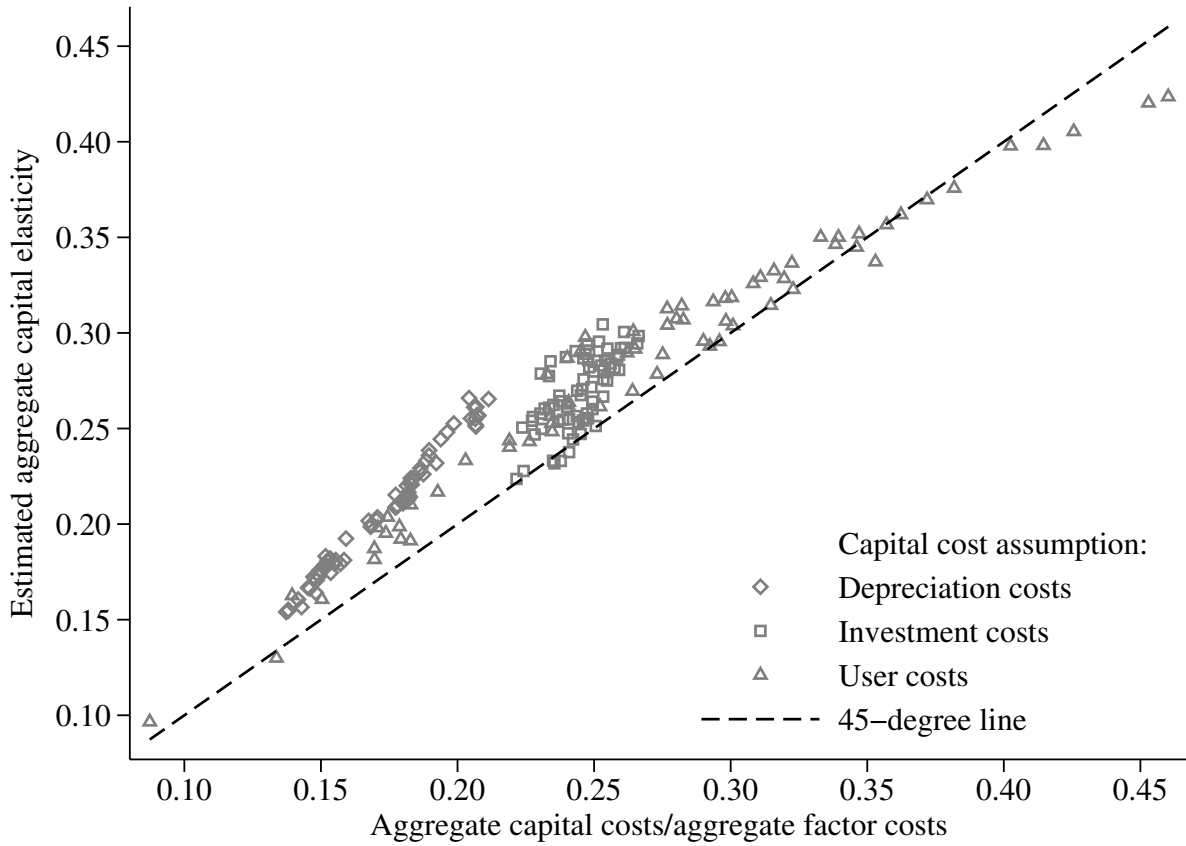
Notes: The estimate of the aggregate capital elasticity, ϵ_K , is made using equation (9) under various assumptions explained in detail in the text. The no-profit assumption assumes capital costs equal all value-added minus labor compensation. The depreciation-only assumption assumes capital costs equal the value of depreciation reported. The primary data source for all estimates is the Bureau of Economic Analysis, with input-output tables, capital stocks by industry, compensation by industry, and value-added by industry using different industrial classifications merged according to a methodology described in the Appendix.

Figure 2: Alternative estimates of the aggregate capital elasticity, ϵ_{Kt} , U.S. 1948-2018



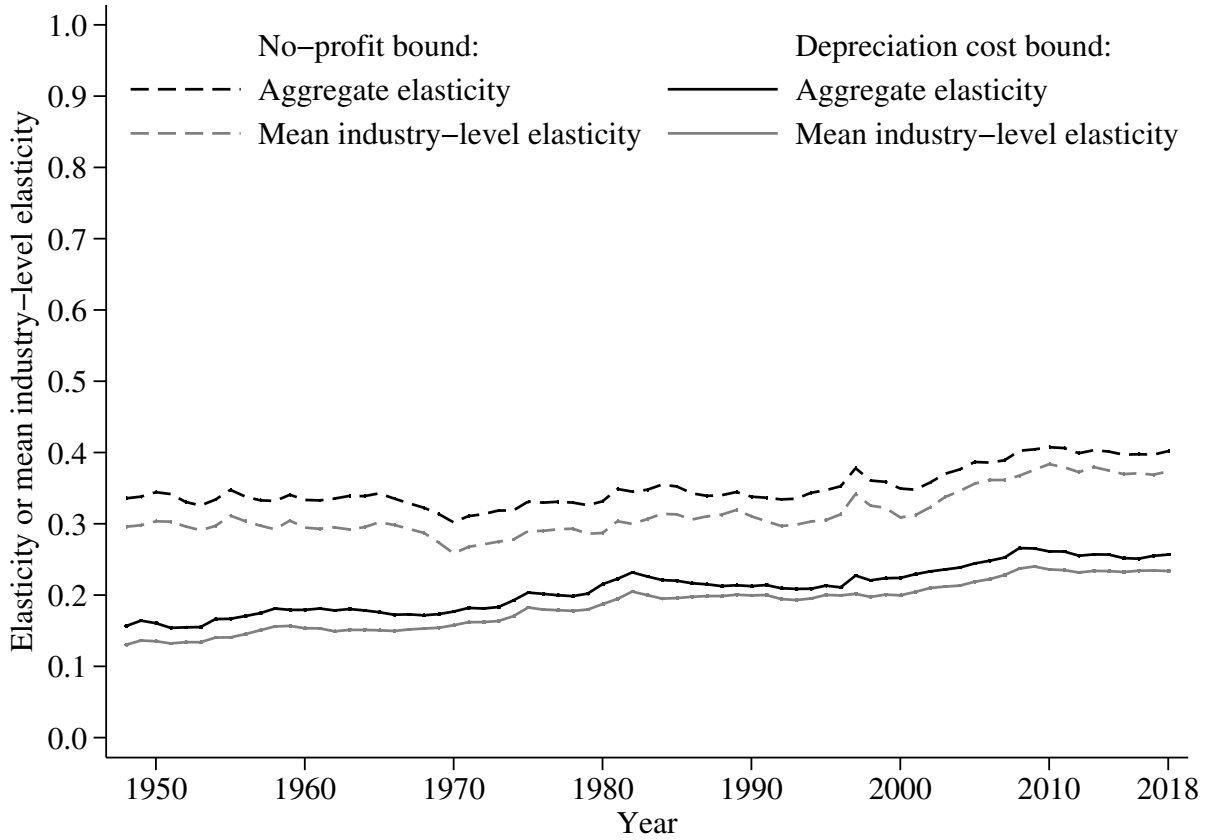
Notes: The estimate of the aggregate capital elasticity, ϵ_K , is made using equation (9) under various assumptions explained in the text. The no-profit and depreciation-only bounds are the same as in Figure 1. The investment cost assumption assumes capital costs equal reported investment, and the user cost assumption assumes capital costs are determined by a standard user cost formula from Hall and Jorgenson (1967). The primary data source for all estimates is the Bureau of Economic Analysis, with input-output tables, capital stocks by industry, compensation by industry, and value-added by industry using different industrial classifications merged according to a methodology described in the Appendix. Additional information on nominal rates of return and inflation rates used for the user cost calculation is from the Federal Reserve.

Figure 3: Comparison of estimated elasticity, ϵ_{Kt} , to cost ratio, s_{Kt}^{Cost}



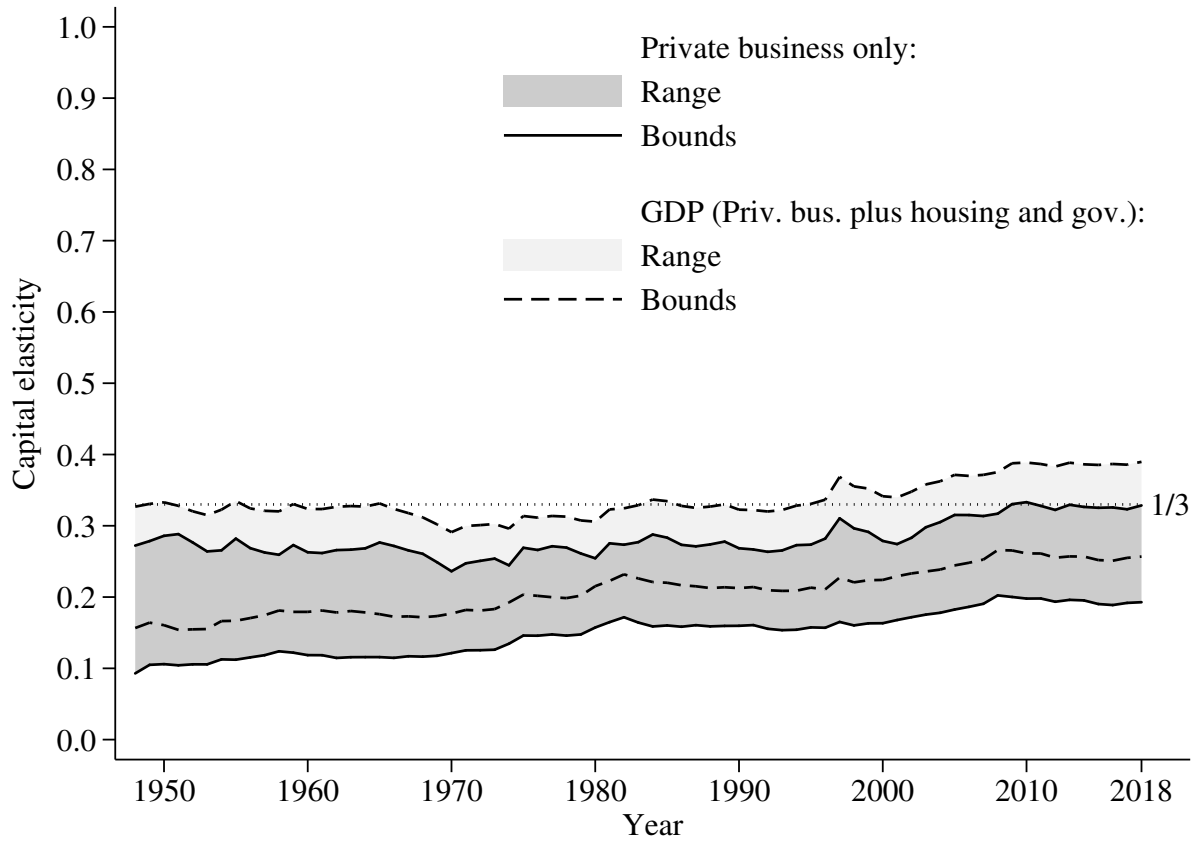
Notes: Estimate of the aggregate capital elasticity, ϵ_{Kt} , are plotted on the y-axis and made using equation (9) under various assumptions explained in the text and denoted in the legend. The cost ratio of capital, s_{Kt}^{Cost} , is calculated as in equation (21) under the same assumptions. The position of the ϵ_{Kt} estimates above the 45-degree line indicates that the input/output relationships matter to some degree, and that industries with high capital elasticities tend to be suppliers to other industries, pulling up the aggregate elasticity.

Figure 4: Aggregate capital elasticity and mean industry-level capital elasticity, U.S. 1948-2018



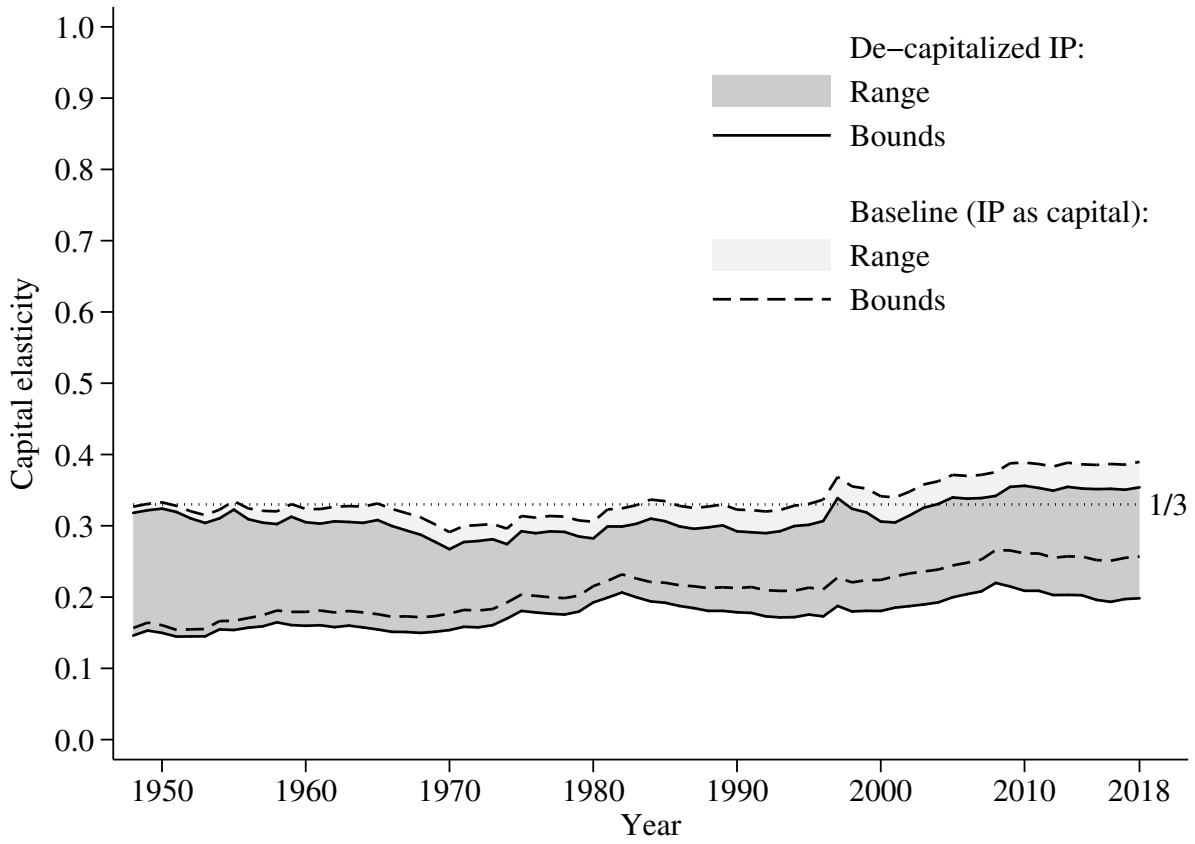
Notes: The estimates of the aggregate capital elasticity (black lines), ϵ_{Kt} , are made using equation (9) in the text. The two estimates differ in the assumption regarding capital costs - depreciation costs only or a no-profit assumption - as explained in the text. The mean industry-level capital elasticity (gray lines) is the term $\bar{\ell}_{Kt}$ from equation (23). It is the raw average of the elements ℓ_{iKt} from the Leontief inverse found in equation (9). The difference between the aggregate elasticity and the mean industry-level elasticity is due to the covariance of the value-added share of an industry and the industry-level elasticity. In both the depreciation and no-profit case, the covariances are positive as the aggregate elasticity lies above the mean industry-level elasticity. The figure shows the trend upward in the elasticity was due to industries, on average, having higher capital elasticities over time, and not due to a change in the covariance of industry size (in value-added terms) and the size of the elasticity.

Figure 5: Estimates of aggregate capital elasticity, private business sector, U.S. 1948-2018



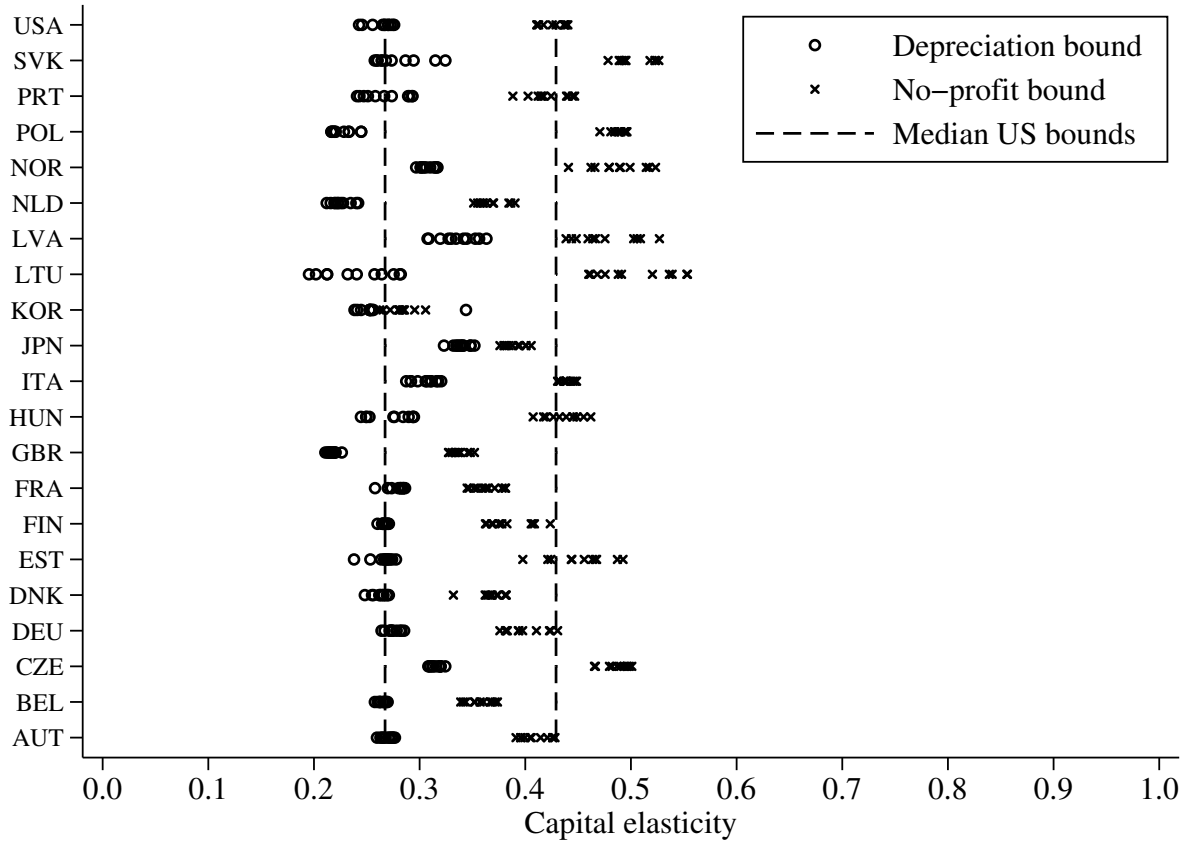
Notes: The estimate of the aggregate capital elasticity, ϵ_{Kt} , is made using equation (9) in the text. Dashed lines refer to the upper (no-profit) and lower (depreciation-only) bounds of ϵ_{Kt} calculated including all industries. The dark lines refer to the upper (no-profit) and lower (depreciation-only) bounds of ϵ_{Kt} calculated for the private business sector (e.g. excluding owner-occupied housing and government).

Figure 6: Estimates of aggregate capital elasticity, de-capitalizing IP, U.S. 1948-2018



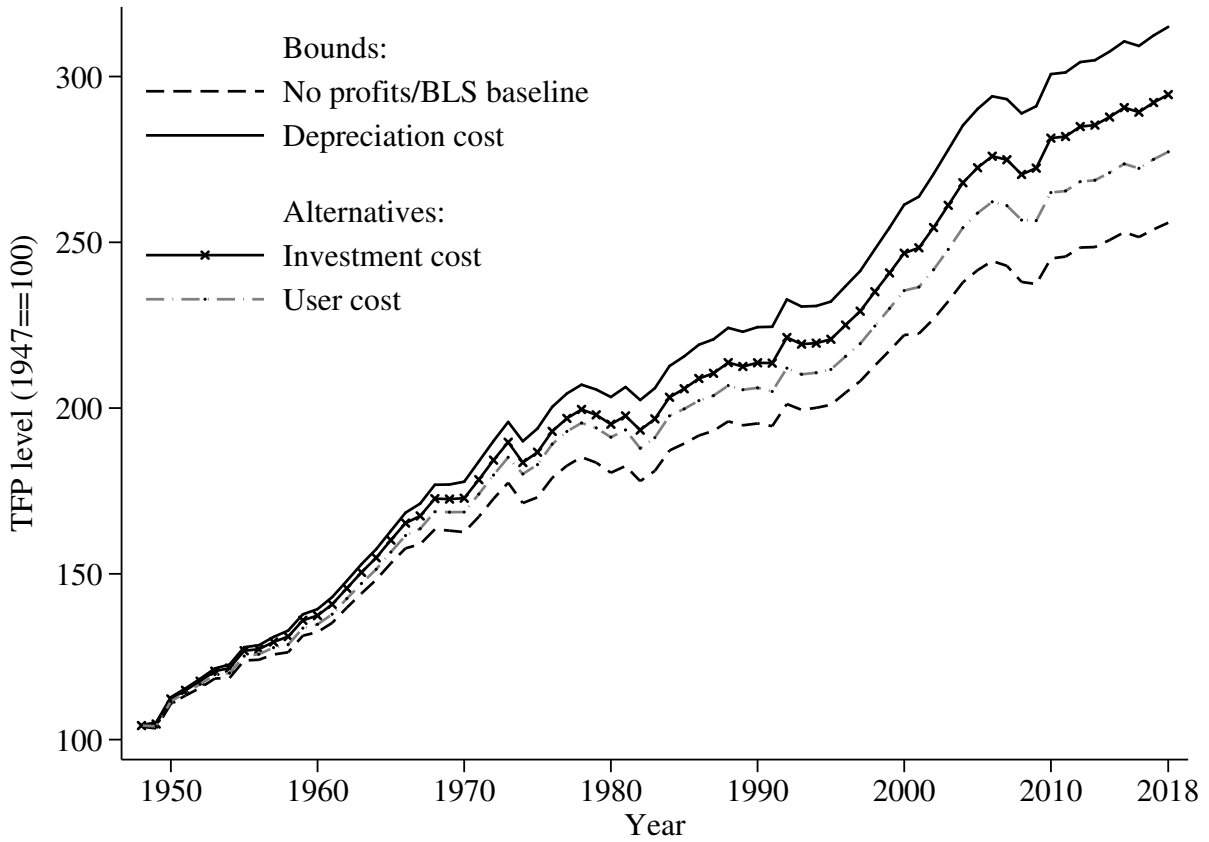
Notes: The estimate of the aggregate capital elasticity, ϵ_{Kt} , is made using equation (9) in the text. Dashed lines refer to the baseline upper (no-profit) and lower (depreciation-only) bounds of ϵ_{Kt} calculated with IP included as a capital good. The dark lines refer to the upper (no-profit) and lower (depreciation-only) bounds of ϵ_{Kt} calculated when IP is de-capitalized from the national accounts as described in the text.

Figure 7: Estimates of aggregate capital elasticity, OECD countries, 2005-2015



Notes: The estimate of the aggregate capital elasticity, ϵ_{Kt} , is made using equation (9) for each OECD country in the sample under two assumptions: no profits (x's) and depreciation costs (o's) only. The U.S. estimates were made using OECD data and assumptions, as described in the text. The dashed lines are medians of the U.S. estimates under depreciation-only and no-profit assumptions.

Figure 8: TFP levels under different elasticity assumptions, U.S. 1948-2018



Notes: The level of TFP is calculated using equation (25) which follows the methodology of the BLS. Data on output, capital services, and labor inputs are from Fernald (2014). The four series plotted differ in the elasticities with respect to capital and labor used in the calculation. Those four series correspond to the four different methods used in this paper to calculate the elasticities, and are based on different assumptions regarding how to calculate the cost of capital by industry. My calculations exclude government and owner-occupied housing, to match the BLS calculations for the private business sector. The “no-profit” elasticities I calculate match the elasticities used by the BLS, and so that series matches the BLS multi-factor productivity numbers with only minor rounding errors.

Table 1: Industrial classification of data by year

Series	I/O tables	Value-added components	Capital stock
1948-62	NAICS 2012 (47 ind)	SIC 1972	BEA/NAICS 2012
1963-86	NAICS 2012 (65 ind)	SIC 1972	BEA/NAICS 2012
1987-96	NAICS 2012 (65 ind)	SIC 1987	BEA/NAICS 2012
1997-2018	NAICS 2012 (71 ind)	NAICS 2012	BEA/NAICS 2012

Notes: This table shows the classifications used for each range of years. The complete mapping of industry data across sources is provided in the Appendix. All data are from the BEA, and described in detail in Section 3.

Table 2: Estimates of U.S. capital elasticity, ϵ_K , under different assumptions

Assumption	Summary statistics, ϵ_{Kt} , 1948-2018:				Fitted change 1948-2018:		
	Mean (1)	Median (2)	Minimum (3)	Maximum (4)	$\Delta\hat{\epsilon}_{K,48-18}$ (5)	Slope ($\hat{\beta}_1$) (6)	R-squared (7)
Panel A: Baseline							
No-profit	0.337	0.328	0.291	0.390	0.069	0.0010	0.572
Investment cost	0.269	0.267	0.224	0.304	0.055	0.0008	0.702
User cost	0.286	0.296	0.097	0.424	0.088	0.0013	0.144
Depreciation cost	0.208	0.211	0.154	0.266	0.106	0.0015	0.920
Panel B: Private business sector							
No-profit	0.282	0.274	0.236	0.333	0.061	0.0009	0.535
Investment cost	0.191	0.195	0.147	0.234	0.076	0.0011	0.789
User cost	0.221	0.228	0.076	0.338	0.081	0.0012	0.223
Depreciation cost	0.150	0.157	0.093	0.202	0.100	0.0014	0.929
Panel C: De-capitalizing intellectual property							
No-profit	0.311	0.305	0.267	0.356	0.045	0.0006	0.327
Investment cost	0.234	0.232	0.210	0.266	0.008	0.0001	0.037
User cost	0.248	0.257	0.066	0.393	0.040	0.0006	0.032
Depreciation cost	0.177	0.178	0.145	0.220	0.059	0.0008	0.747
Panel D: Private business sector and decapitalizing IP							
No-profit	0.258	0.254	0.219	0.297	0.024	0.0003	0.145
Investment cost	0.157	0.154	0.136	0.201	0.012	0.0002	0.052
User cost	0.182	0.181	0.050	0.307	0.020	0.0003	0.017
Depreciation cost	0.120	0.121	0.086	0.150	0.040	0.0006	0.585

Notes: The calculation of ϵ_{Kt} is described in the text. The panels of the table refer to different assumptions made regarding the inclusion/exclusion of owner-occupied housing and intellectual property capital (the baseline includes both). Within the panel, the rows refer to assumptions about the capital costs by industry, $COST_{iKt}$, that are used to calculate ϵ_{Kt} . The specifics of those assumptions are discussed in the text. The fitted change is estimated from a simple OLS regression of ϵ_{Kt} against time for the given variant, with $\hat{\beta}_1$ showing the estimated change per year, and $\Delta\hat{\epsilon}_{K,48-18} = 70 \times \hat{\beta}_1$ being the estimated overall change from 1948 to 2018. The R-squared is from the simple OLS regression.

Table 3: Capital costs as share of factor costs and value-added, by sector

Variant	Summary statistics, 1948-2018:							
	Capital costs/Factor costs, s_{Kt}^{Cost}				Capital costs/Value-added, s_{Kt}^{VA}			
	Mean (1)	Median (2)	Minimum (3)	Maximum (4)	Mean (5)	Median (6)	Minimum (7)	Maximum (8)
Panel A: All industries								
No-profit	0.337	0.328	0.291	0.390	0.337	0.328	0.291	0.390
Investment cost	0.245	0.246	0.222	0.266	0.215	0.217	0.183	0.242
User cost	0.272	0.275	0.087	0.460	0.258	0.250	0.067	0.576
Depreciation cost	0.174	0.178	0.137	0.211	0.139	0.145	0.107	0.164
Panel B: Private business sector								
No-profit	0.282	0.274	0.236	0.333	0.282	0.274	0.236	0.333
Investment cost	0.176	0.177	0.139	0.208	0.153	0.154	0.116	0.185
User cost	0.213	0.213	0.072	0.346	0.198	0.189	0.059	0.385
Depreciation cost	0.134	0.141	0.088	0.174	0.111	0.120	0.070	0.141

Notes: The cost ratios are calculated as in equations (21) and (22). The panels of the table refer to different sectors of the economy. Panel A includes all industries. Panel B is just the private business sector, which excludes owner-occupied housing and government. Owner-occupied housing refers to NAICS codes HS, ORE, and 531. Government refers to NAICS codes GFGD, GFGN, GFE, GSLG, GSLE, and GFG, which covers federal, state, and local government, both general and enterprises. In each row, the assumption made to calculate capital costs is labeled, as described in the text. Columns (1)-(4) are summary statistics over 1948-2018 for the total estimated capital costs divided by total factor costs (the sum of capital costs and labor costs). Columns (5)-(9) are summary statistics over 1948-2018 for total capital costs divided by value-added.

Table 4: Estimates of capital elasticity, ϵ_K , for OECD countries 2005-15

Country	Mean 2005-15 $\bar{\epsilon}_K$:			Change $\Delta\epsilon_{K,05-15}$:		
	Capital cost assumption:			Capital cost assumption:		
	Depreciation lower bound (1)	Investment (2)	No-profit upper bound (3)	Depreciation lower bound (4)	Investment (5)	No-profit upper bound (6)
Austria	0.276	0.319	0.428	0.005	-0.008	-0.032
Belgium	0.273	0.296	0.387	-0.001	-0.006	-0.002
Czech Republic	0.322	0.372	0.503	0.006	-0.021	-0.006
Denmark	0.264	0.287	0.373	-0.023	-0.022	-0.013
Estonia	0.268	0.349	0.458	-0.000	-0.024	-0.069
Finland	0.273	0.294	0.402	0.004	-0.008	-0.032
France	0.281	0.312	0.371	0.027	0.012	-0.015
Germany	0.278	0.303	0.409	-0.010	-0.012	-0.036
Hungary	0.275	0.320	0.442	0.031	0.015	0.051
Italy	0.325	0.340	0.490	0.021	-0.035	-0.020
Japan	0.345	0.315	0.409	0.013	0.022	-0.015
Latvia	0.339	0.363	0.487	-0.003	-0.105	-0.064
Lithuania	0.245	0.311	0.518	0.016	-0.047	-0.018
Netherlands	0.230	0.270	0.389	-0.015	0.026	-0.005
Norway	0.309	0.364	0.493	-0.018	-0.048	-0.078
Portugal	0.275	0.303	0.449	0.049	-0.027	0.045
Slovakia	0.285	0.324	0.521	-0.066	-0.025	-0.036
South Korea	0.290	0.235	0.411	-0.099	-0.071	-0.026
United Kingdom	0.221	0.243	0.359	-0.000	-0.009	-0.009
United States (OECD)	0.266	.	0.431	0.022	.	0.017
OECD mean	0.280	0.313	0.441	-0.003	-0.020	-0.018
OECD median	0.276	0.312	0.433	0.005	0.000	0.005
United States (BEA)	0.256	0.291	0.381	0.008	-0.005	0.014

Notes: The calculation of ϵ_{Kt} is described in the text. The value of ϵ_{Kt} is calculated under three different assumptions: depreciation costs only, investment cost, and no-profits. Each is described in more detail in the text. Means in columns (1)-(3) are across the years 2005-15 for a given country. The differences in columns (4)-(6) are $\epsilon_{K,2015} - \epsilon_{K,2005}$. The two OECD rows show the mean and median, respectively, across all years and all countries listed above those rows. The differences in the OECD rows refer to differences in those means and medians, respectively. The “United States (OECD data)” row uses data from STAN and the OECD, the same source as for the other OECD countries. Information on investment (gross fixed capital formation) was not available in STAN for the U.S., and hence those estimates of ϵ_{Kt} are missing. The “United States (BEA data)” row uses data directly from the BEA, as described in the earlier section of the paper. Differences in the OECD and BEA calculations for the U.S. are, in part, due to differences in treatment of proprietors income.

Table 5: Average annual TFP growth (%), by capital cost assumption

Years	Assumption on capital costs:			
	No-profit (BLS) (1)	User cost (2)	Investment cost (3)	Depreciation cost (4)
1950-1959	1.89	2.01	2.13	2.22
1960-1969	2.31	2.49	2.53	2.65
1970-1979	1.35	1.56	1.51	1.61
1980-1989	0.85	0.80	0.95	1.03
1990-1999	1.18	1.22	1.33	1.39
2000-2009	0.74	0.95	1.10	1.20
2010-2018	0.54	0.57	0.57	0.58
1948-2018	1.29	1.40	1.48	1.58

Notes: All growth rates reported in percents. TFP growth is calculated using equation (25) to find annual growth rate, which is then converted to a level of TFP using 1948 as a base = 100. Average annual growth rates are calculated using those levels of TFP. This is done for the private business sector only (excluding government and housing) to match BLS procedures. The different assumptions on capital costs correspond to the s parameter in equation (25) and refer to different assumptions about capital costs used to calculate ϵ_{Kt}^s and ϵ_{Lt}^s . The “no-profit” capital cost assumption in column (1) is equivalent to the BLS assumption regarding elasticities.