The Elasticity of Aggregate Output with Respect to Labor and Capital

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Research questions

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- What are the elasticities of GDP with respect to capital and labor?
- Have those elasticities changed over time?

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- Have those elasticities changed over time?

Our “rule of thumb” is that capital elasticity is $\alpha = 1/3$:

- Presumes a coherent aggregate production function
- Is based on labor’s share of total GDP; presumes zero profits

Why do we care?

The answer informs us on:

- Relative importance of labor and capital as factors
- Convergence speed, transition dynamics
- Distribution of GDP to labor, capital, profits
- TFP growth rate
- Macro calibrations

This paper

Estimate elasticities with looser assumptions:

- Industry-specific elasticities linked through input/output network
- Allow for arbitrary market power at industry level
- Bound the estimates given issues in measuring capital costs
- Applies methodology in Baqaee and Farhi (2017, 2018)

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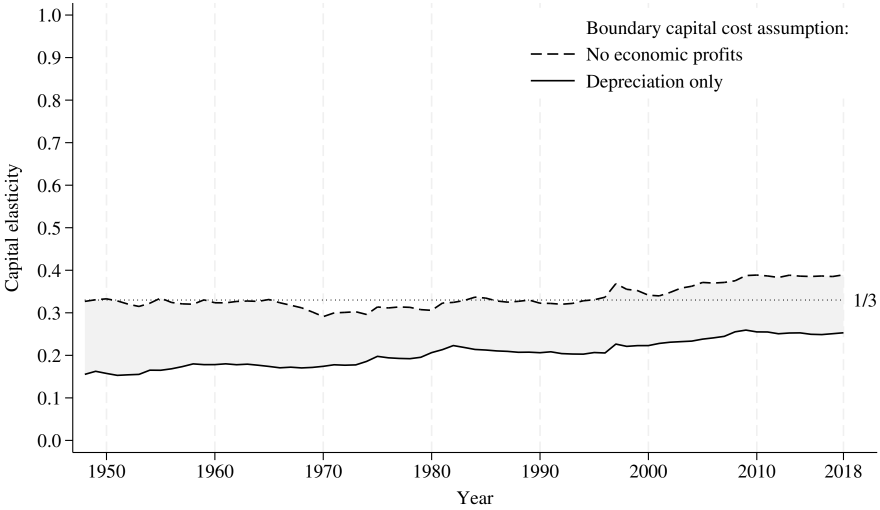
- Industry-specific elasticities linked through input/output network
- Allow for arbitrary market power at industry level
- Bound the estimates given issues in measuring capital costs
- Applies methodology in Baqaee and Farhi (2017, 2018)

Scope of work:

- Provide estimates for US 1948-2018
- Evaluate influence of industry and capital types (e.g. IP, housing)
- Re-assess estimates of TFP growth 1948-2018

Preview of results

Robert Solow was kind-of, sort-of right?



Why do we care?

The answer informs us on:

- Relative importance of labor and capital as factors $\rightarrow$ labor matters more?
- Convergence speed, transition dynamics $\rightarrow$ shocks dissipate faster?
- Distribution of GDP to labor, capital, profits $\rightarrow$ profits lowered labor share?
- TFP growth rate $\rightarrow$ higher, but bigger swings in 1990s/2000s
- Macro papers calibrated using $\alpha = 1/3$ $\rightarrow$ kinda right?

Relevance and contribution

Literature on labor's share of GDP: Gollin (2002); Young and Zuleta (2013a,b); Elsby, Hobijn, and Sahin (2013); Karabarbounis and Neiman (2014); Gomme and Rupert (2014); Rognlie (2015); Barkai (2017); Smith, Yagan, Zidar, Zwick (2017); Karabarbounis and Neiman (2018); Koh, Santaeulalia-Llopis, Zheng (2018)

Differences and similarities:

- Elasticities don't equal shares if markups > 1
- Elasticities could provide part of explanation for labor share decline
- Elasticity calculation explicitly at industry level vs. aggregate
- Same data and imputation problems

Theoretical Roadmap

Capital elasticity (same logic for labor elasticity) is ϵ_K:

- ϵ_K is weighted sum of industry-level capital elasticities
- Industry-level capital elasticities are inferred from capital cost shares (industry-level cost minimization, Shepherd's lemma)
- Weights reflect both industry's share of final use and their share of *costs* in other industries (I/O relationships)

Theoretical setting

Borrowed from Baqaee and Farhi (2017, 2018)

Each industry i has constant-returns cost function. Industry i has costs as follows:

$$COST_i = COST_{iM} + COST_{iK} + COST_{iL} \tag{1}$$

The first term is total intermediate costs from J total industries:

$$COST_{iM} = \sum_{j=1}^{J} COST_{ij} \tag{2}$$

Theoretical setting

Cost shares for intermediates defined as

$$\lambda_{ij} = \frac{COST_{ij}}{COST_i} \tag{3}$$

and for factors of production as

$$\lambda_{iK} = \frac{COST_{iK}}{COST_i} \tag{4}$$

$$\lambda_{iL} = \frac{COST_{iL}}{COST_i}. \tag{5}$$

Theoretical setting

Build a matrix of intermediate cost shares

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1J} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2J} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{J1} & \lambda_{J2} & \cdots & \lambda_{JJ} \end{bmatrix} \tag{6}$$

and a matrix of factor cost shares

$$B = \begin{bmatrix} \lambda_{1K} & \lambda_{1L} \\ \lambda_{2K} & \lambda_{2L} \\ \vdots & \vdots \\ \lambda_{JK} & \lambda_{JL} \end{bmatrix} \tag{7}$$

Theoretical setting

Final use shares of GDP. Let $GDP = \sum_{j=1}^{J} F_j$, where F_j is final use of j, then

$$\gamma_j = \frac{F_j}{GDP}. \tag{8}$$

Collect in a vector,

$$\Gamma' = \begin{bmatrix} \gamma_1 & \gamma_2 & \cdots & \gamma_J & \end{bmatrix} \tag{9}$$

Theoretical setting

Baqaee and Farhi show that elasticities with respect to capital (ϵ_K) and labor (ϵ_L) are as follows:

$$\begin{bmatrix} \epsilon_K & \epsilon_L \end{bmatrix} = \Gamma'(I - \Lambda)^{-1} B \tag{10}$$

Theoretical intuition

Look at capital elasticity to break down:

$$\epsilon_K = \sum_j^J \left(\sum_i^J \gamma_i \ell_{ij} \right) \lambda_{jK} \tag{11}$$

- γ_i: dollars of i in final use
- ℓ_{ij}: from Leontief inverse $(I - \Lambda)^{-1}$, cost to j of producing one dollar of i given all I/O relationships
- λ_{jK}: share of costs of j that are used on capital (capital elasticity for j)

The weight on each λ_{jK} is cost to j of producing all final use

Theoretical setting

This nests your favorite methods for estimating ϵ_K and ϵ_L.

- Solow (1957): $\lambda_{ij} = 0$ (no I/O) and $\lambda_{iK} = RK_i/VA_i$ (zero profits)
- Hulten (1978): $\lambda_{ij} \neq 0$ (I/O) and $\lambda_{iK} = RK_i/VA_i$ (zero profits)
- Hall (1988, 1990): $\lambda_{ij} = 0$ (no I/O) and $\lambda_{iK} = RK/(RK + wL)$ (profits)

Implementation

Big issues in plugging data into the Baqaee and Farhi structure given national accounts. Simplifying

$$VA = COMP + TAX + PROP + ROS \tag{12}$$

Cannot cleanly extract labor or capital costs for any industry

1. Proprietors income ($PROP$) contains labor costs, capital costs, economic profits
2. Residual operating surplus (ROS) contains capital costs and economic profits

Problem measuring costs ....

Implementation

....and the industry definitions are not consistent over time.

Series	I/O table	National accounts	Capital Stock
1947-62	NAICS 2012 (47 ind)	SIC 1972	BEA/NAICS 2012
1963-86	NAICS 2012 (65 ind)	SIC 1972	BEA/NAICS 2012
1987-96	NAICS 2012 (65 ind)	SIC 1987	BEA/NAICS 2012
1997-18	NAICS 2012 (71 ind)	NAICS 2012	BEA/NAICS 2012

All data is from the BEA. Used BEA crosswalks and own assumptions to map all data into NAICS 2012 coding that matched the I/O tables.

Matching

Implementation

Absence of precise cost information for capital and labor. Strategy:

- I/O table reports actual costs of intermediates (sort of)
- Allocate proprietors income to calculate labor costs
- Construct different series of ϵ_K and ϵ_L based on capital cost assumptions
- Try to *bound* the elasticities based on theory/data
- Undertake variations on assumptions

I/O costs

Implementation

Labor costs: Allocate a portion of proprietors income to labor. General principle:

$$COST_{iLt} = COMP_{it} + PROP_{it} \left(\frac{COMP_{it}}{VA_{it} - PROP_{it}} \right). \tag{13}$$

This follows Gomme and Rupert (2004).

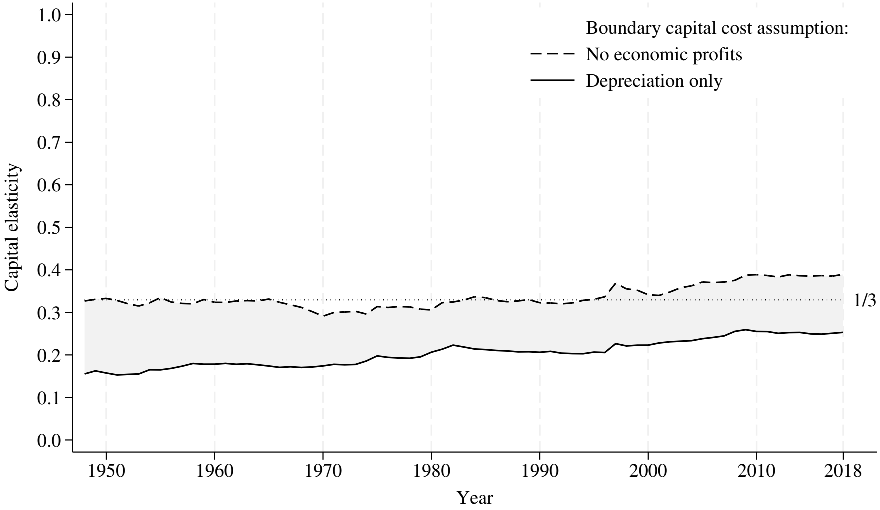
No-profit upper bound

Capital costs: Set the *upper bound* for capital costs by assuming there are zero profits.

$$COST_{iKt}^{NoProf} = VA_i - COST_{iLt}. \tag{14}$$

Gives an *upper bound* for ϵ_K. Note this will be the *lower bound* for ϵ_L.

No-profit upper bound



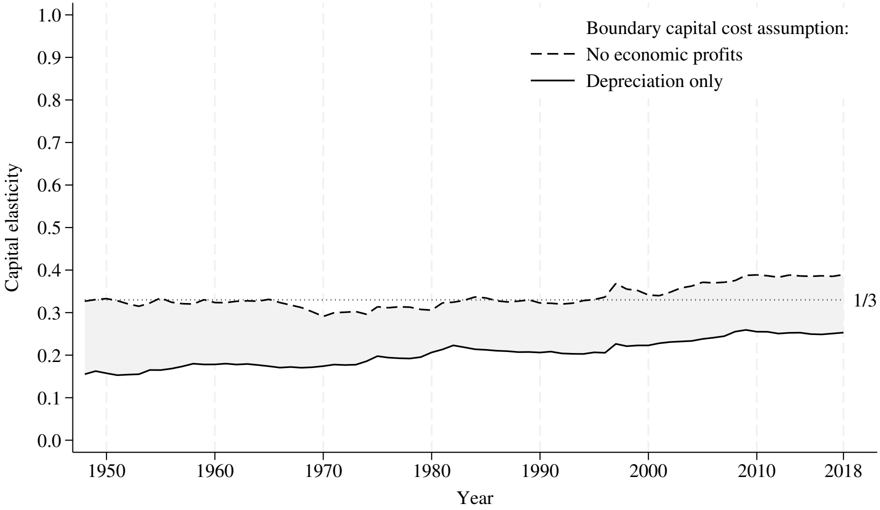
Depreciation lower bound

Capital costs: Set the *lower bound* for capital costs by using the cost of depreciation ($DEPR_{it}$), which is reported by industry. Assumes zero financing costs of existing capital stock.

$$COST_{iKt}^{Depr} = DEPR_{it}. \tag{15}$$

Gives an *lower bound* for ϵ_K. Note this will be the *upper bound* for ϵ_L.

Depreciation lower bound



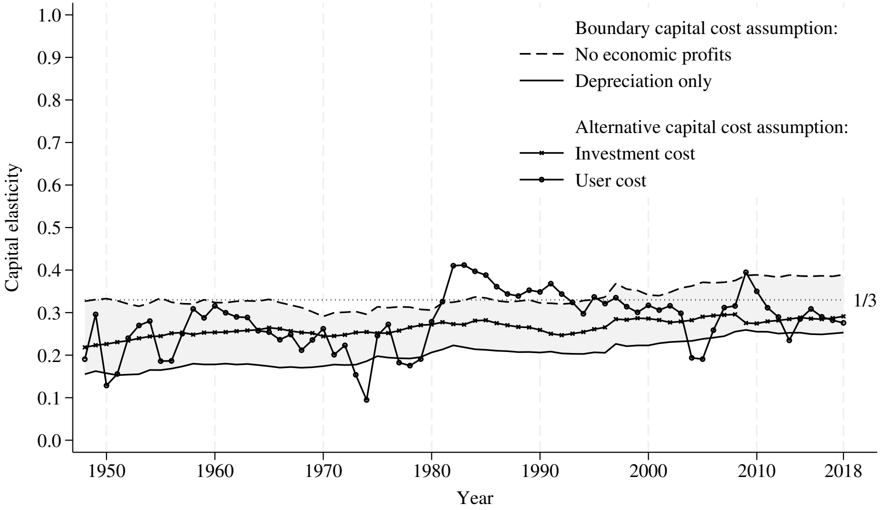
Alternative estimates

Capital costs: Total investment (INV_{it}) is reported by industry. Combines replacement of depreciation and purchase of new capital goods. In Golden rule world $INV = RK$.

$$COST_{iKt}^{Inv} = INV_{it}. \tag{16}$$

Calculate ϵ_K and ϵ_L.

Alternative estimates



Alternative estimates

Capital costs: Calculate the user cost of capital by industry. Three types of capital (structure, equipment, IP).

$$COST^{User}_{iKt} = \sum_{j \in st,eq,ip} K_{ijt} R_{ijt}. \tag{17}$$

where

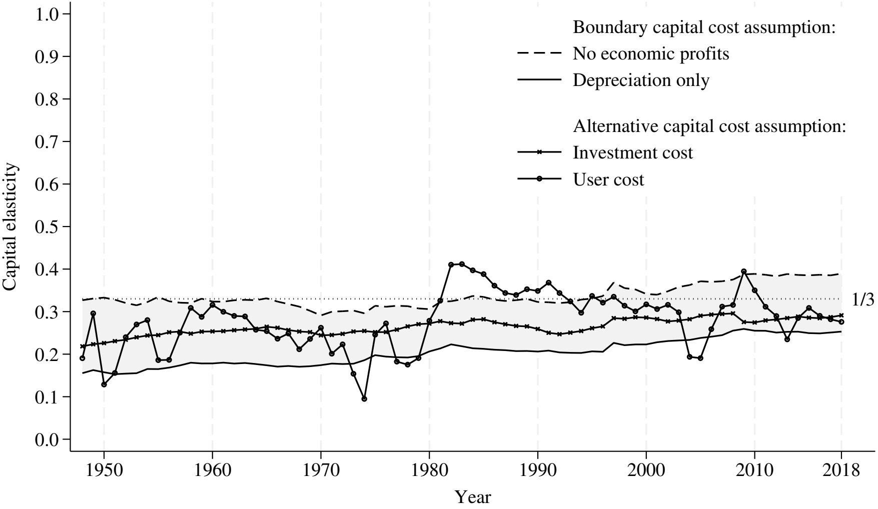
$$R_{ijt} = (Int_{it} - E[\pi_{ijt}] + \delta_{ijt}) \frac{1 - z_{jt}\tau_t}{1 - \tau_t} \tag{18}$$

is the rental rate of each type.

- Int_{it}: nominal interest rate facing industry i
- $E[\pi_{ijt}]$: expected inflation of capital type j for industry i
- δ_{ijt}: depreciation of capital type j for industry i (BEA)
- z_{jt}: depreciation allowance for capital type j in tax code (BEA)
- τ_t: effective corporate tax rate (BEA)

Details

Alternative estimates



Robustness

Can alter the general approach in the following ways and get very similar results (i.e. differ in third decimal place):

- Use "After Redefinitions" I/O tables that re-assign some transactions (1997-2018).
- Excluded imported intermediates from I/O tables (1997-2018).
- Do not allow negative costs for capital (1948-2018).
- Use different assumptions on proprietors income (1948-2018, bigger differences).

Aggregate cost shares

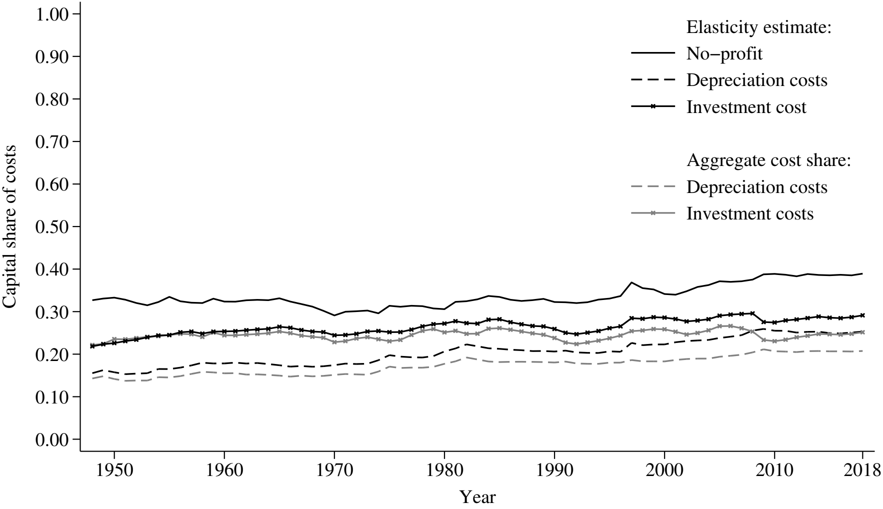
Define the following aggregate cost share for capital:

$$s_{Kt}^{Cost} = \frac{\sum_{j \in J} COST_{jKt}}{\sum_{j \in J} COST_{jKt} + COST_{jLt}}. \tag{19}$$

If there are zero profits, then $\epsilon_{Kt} \to s_{Kt}^{Cost}$.

If $\epsilon_{Kt} > s_{Kt}^{Cost}$, markups skew costs towards low capital cost industries

Aggregate cost shares



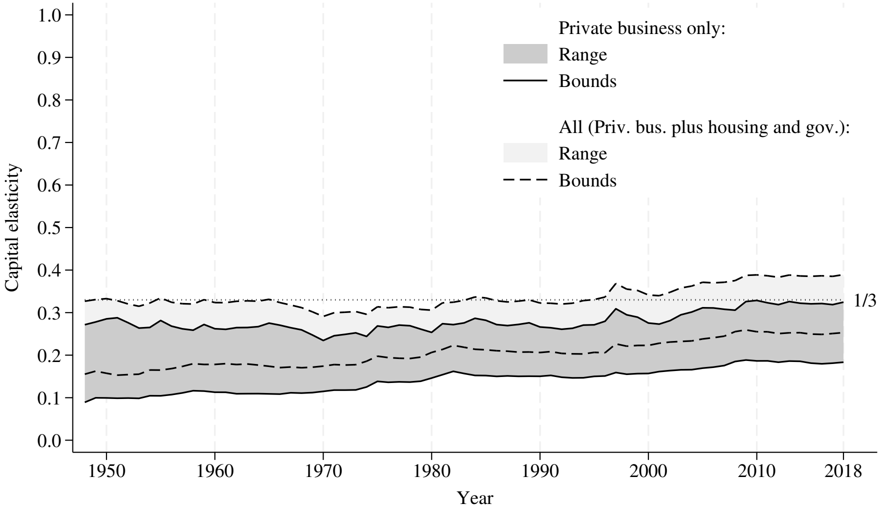
Private business sector

Private sector business only:

- Exclude government (cost shares not far from average)
- Exclude housing (relatively high capital and low labor cost)

Lower implied capital elasticity (and higher labor elasticity)

Private business sector



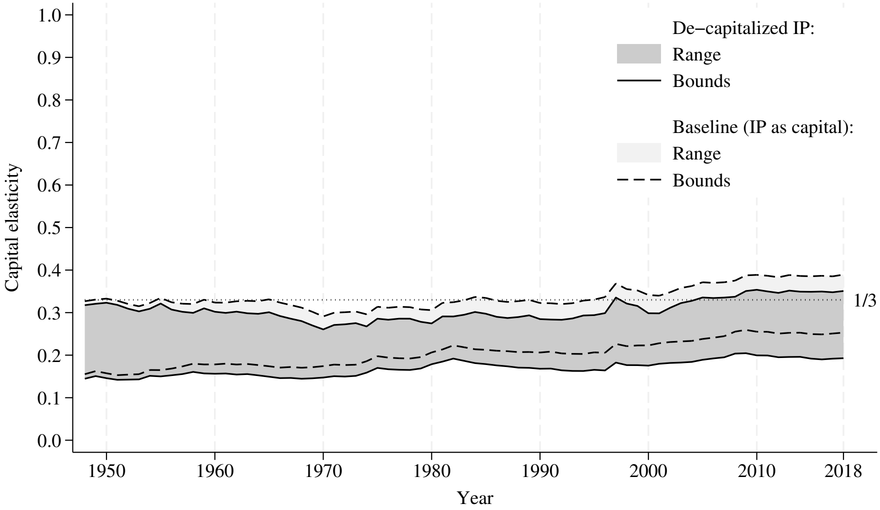
IP Adjustment

Intellectual property?

- Elasticity rises over time
- But may be because data on IP from pre-1990 is scarce?
- Koh, Santaeulalia-Llopis, Zheng (2018): aggregate labor share falls due to IP accounting

Details

IP Adjustment



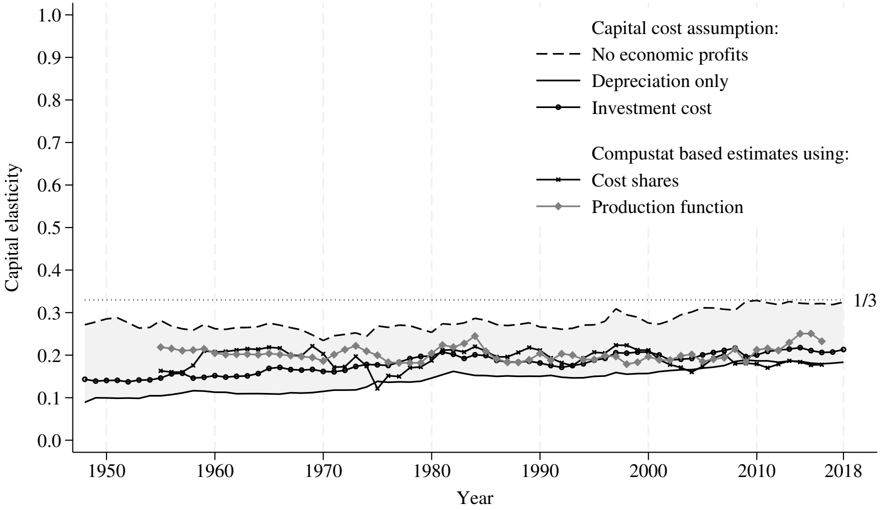
Firm-level data

De Loecker, Eeckhout, and Unger (2020) use Compustat to infer/estimate cost shares of firms:

- Use firms within an industry i to set industry-level cost shares, λ_{iK}
- Use income statement information or production function estimates
- I use their estimates to calculate ϵ_K and ϵ_L
- Not firm-level estimates, estimates consistent with Compustat firm-level data
- Compare to private business sector estimates

Details

Firm-level data



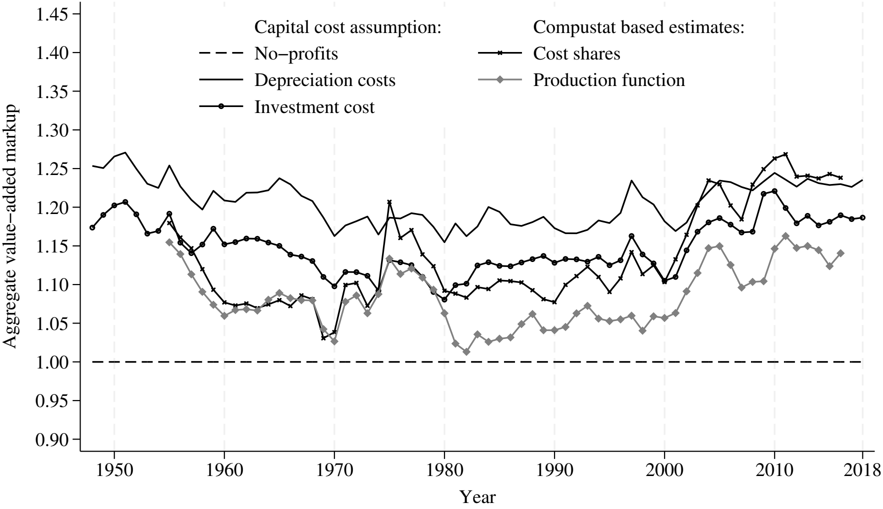
Market power and the labor share

Different assumptions about capital costs imply different levels of profits and markups:

$$\mu_t^{VA} = \frac{\sum_{j=1}^{J} VA_{jt}}{\sum_{j=1}^{J} COST_{jKt} + COST_{jLt}}. \tag{20}$$

What do these assumptions about capital costs imply about markups?

Market power and the labor share



Market power and the labor share

Did labor's share of GDP fall because ϵ_K went up (ϵ_L went down) or because profits went up?

- Bounds on ϵ_K do appear to rise near end of period
- Series based on investment costs, Compustat, show no trend in ϵ_K
- Lean towards profit explanation, consistent with markups?

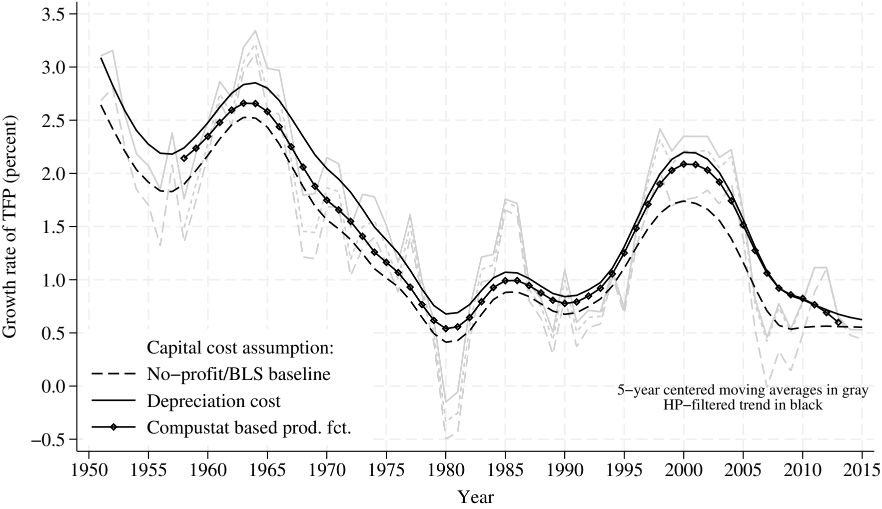
Growth Accounting

For a typical growth accounting exercise:

$$d \ln TFP_t = d \ln Y_t - \epsilon_{Kt} d \ln K_t - \epsilon_{Lt} d \ln L_t. \tag{21}$$

- The implied growth in TFP depends on the elasticities
- BLS uses the “no-profit” assumption only
- Effect is ambiguous

Growth Accounting



Growth Accounting

	Assumption on capital costs:				
	National accounts only:			Compustat derived:	
Years	No-profit (1)	Invest. cost (2)	Depr. cost (3)	Prod. fct. (4)	Cost shares (5)
1950-1959	1.89	2.15	2.24	1.60	1.75
1960-1969	2.31	2.55	2.67	2.46	2.44
1970-1979	1.35	1.53	1.63	1.46	1.57
1980-1989	0.85	0.97	1.04	0.96	0.96
1990-1999	1.19	1.34	1.41	1.35	1.31
2000-2009	0.76	1.14	1.24	1.17	1.21
2010-2018	0.54	0.57	0.58	0.45	0.46
1948-2018	1.29	1.51	1.60	1.34	1.36

Conclusions

Revisiting results:

- The rule-of-thumb capital elasticity (1/3) is an upper bound
- Realistic assumptions lower estimate (IP, housing, markups)
- Profits, not elasticities, explain decline in labor share (?)
- TFP growth is higher on average, more dramatic swings in 1990-2018 period

More broadly, labor is “more important” than normally assumed

Matching

$ELEM_{it}^{NAICS}$ is some element (e.g. labor compensation, depreciation) that I need for a NAICS industry i, but only reported on SIC basis as $ELEM_{jt}^{SIC}$ for industry j. General concept is this:

$$ELEM_{it}^{NAICS} = VALU_{it}^{NAICS} \times \frac{ELEM_{jt}^{SIC}}{VALU_{jt}^{SIC}}. \tag{22}$$

Still requires linking a NAICS industry to an appropriate SIC industry(s). $VALU_{it}^{NAICS}$ and $VALU_{jt}^{SIC}$ are available in all cases.

Matching

From easy to hard:

- **One SIC to one NAICS:** 1947-62 the SIC industry “Construction” (SIC 1972 code C) is matched to NAICS industry “Construction” (NAICS code 23).
  - Apply formula.
- **One SIC to many NAICS:** 1947-62 the SIC industry “Retail trade” (SIC code G) is matched to NAICS industries “Retail trade” (NAICS code 44RT) and “Food service and drinking places” (NAICS code 722).
  - Apply same SIC ratio $ELEM_{jt}^{SIC}/VALU_{jt}^{SIC}$ to multiple NAICS industries.

Matching

From easy to hard:

- **Many SIC to one NAICS:** 1947-62 “Banking” (SIC code 60), “Credit agencies” (SIC code 61), “Security and commodity brokers” (SIC code 62), “Insurance carriers” (SIC code 63), and “Insurance agents, brokers” (SIC code 64) all being matched to NAICS industry “Finance and Insurance” (NAICS code 52).
  - Sum SIC industry $ELEM_{jt}^{SIC}$ and sum SIC industry $VALU_{jt}^{SIC}$ and then calculate $ELEM_{jt}^{SIC}/VALU_{jt}^{SIC}$, apply to NAICS industry.

Return

From Make/Use to Cost Shares

BEA I/O tables distinguish J industries from M commodities, although for most practical purposes these align (e.g. agriculture industry produces the agricultural commodity), but not exactly. Commodities like “used/scrap” or “noncomparable imports” exist.

- **Use Table:** U, is a $M \times J$ matrix. u_{mj} shows the amount of a commodity m used as an input by industry j
- **Make Table:** V, is a $J \times M$ matrix. v_{jm} shows the amount produced by industry j of commodity m
- X_M measure the gross output of each of the M commodities (M by 1)
- X_I measure the gross output of each of the J industries (J by 1)
- F_I measure the final use of each of the J industries (J by 1)

From Make/Use to Cost Shares

Let

$$A = V\hat{X}_M^{-1} \tag{23}$$

A is a $J \times M$ matrix. a_{im} measures the share of gross output of commodity m that is produced by industry i. Let

$$C = AU = V\hat{X}_M^{-1}U. \tag{24}$$

C is a $J \times J$ matrix. c_{ij} is the spending by industry j on output of industry i (working through the commodities used by j and produced by i).

C is the matrix of costs used in the calculation of the elasticities

From Make/Use to Cost Shares

Confirm that this logic and calculation is sound. Let:

$$X_I = Ce + F_I \quad (25)$$
$$X_I = C'e + V_I \quad (26)$$

where e is a $Jx1$ vector of 1's, meaning gross output is the sum of intermediate sales and final use sales, or gross output is the sum of intermediate purchases plus value added.

Given C, F_I, X_I, can solve for V_I, industry value-added. Short of small rounding errors this is equal to BEA reported value-added, confirming calculations.

Return

Implementation

...and Int_{it} defined as

$$Int_{it} = \sum_m s_{imt} Int_{mt} \tag{27}$$

- s_{imt} shares of financing from source m (e.g. mortgages, corp bonds). From Fed Flow of Funds.
- Int_{mt} are source-specific interest rates (e.g. 30-year mortgage rate, 30-year AAA bond rate). From Fed.

Implementation

...and $E[\pi_{ijt}]$ is

- Proxied with average inflation over following three years.
- BEA reports industry-specific, i, capital-type specific, j, price indices
- Variations (backward looking, 5-year) give similar results

Return

Excluding IP

To exclude IP as a capital cost, have to remove own-account IP spending and accumulation:

- Remove from value-added: $VALU_{it}^{NoIP} = VALU_{it} - INV_{i,IP,t}$
- Remove from investment spending: $INV_{it}^{NoIP} = INV_{it} - INV_{i,IP,t}$
- Remove from depreciation: $DEPR_{it}^{IP} = DEPR_{it} - DEPR_{i,IP,t}$
- Remove from capital stock: $K_{it}^{NoIP} = K_{it} - K_{i,IP,t}$

What this does not account for is IP capital purchased from other industries. So adjustment for IP should be larger.

Return

Compustat

Two types of estimates from De Loecker, Eeckhout, Unger (2020)

- **Cost data:**
  - DLEU calculate capital costs from firm level data.
  - DLEU calculate Non-capital costs are COGS and SGA from firm-level data.
  - I calculate ratio of capital to non-capital costs for firms in given industry.
  - Multiply that firm-derived ratio by sum of intermediate and labor costs for NAICS industry to get NAICS industry level capital costs.
- **Production function:**
  - DLEU estimate industry-level elasticities w.r.t. capital, COGS, and SGA using firm-level data.
  - I calculate ratio of capital elasticity to COGS and SGA elasticities.
  - Multiply that ratio by sum of intermediate and labor costs for NAICS industry to get NAICS industry level capital costs.

Return