# Population and the origin of sustatined growth

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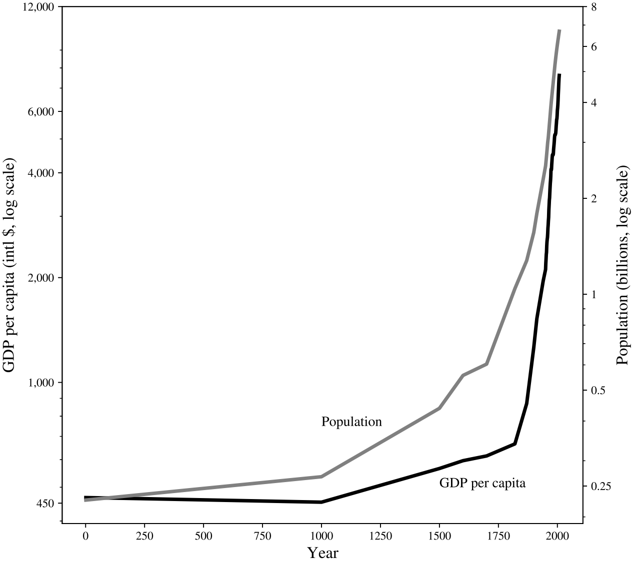
Introduction to Economic Growth

# Historical take-off



# A Malthusian economy

Malthus speculated on how fixed/limited resources influenced population growth and living standards. Let production be

$$Y_t = X^\beta \left( A_t^{\beta/(1-\beta)} L_t \right)^{1-\beta}$$

where $X$ is the amount of the fixed resource. $\beta$ is how important that is in production. $L_t$ is population. $A_t$ is

productivity. The exponents are for simplification.

# Living standards



In per-capita terms this is

$$y_t = \left(\frac{A_t X}{L_t}\right)^{\beta}. \tag{1}$$

Living standards depend

- positively on $A_t$
- positively on $X_t$
- *negatively* on $L_t$

# Endogenous population growth



For Malthus, population growth isn't given, it depends on $y_t$

$$g_L = \nu(y_t - \overline{c})$$

where $\nu$ is scaling and $\overline{c}$ is a "subsistence" level of consumption:

- If $y_t > \overline{c}$, population growth is positive. Lower mortality, higher family formation and fertility.
- If $y_t < \overline{c}$, population growth is negative. High mortality, limited family formation and fertility.
- As $y_t$ goes up, so does $g_L$

# Dynamics of population



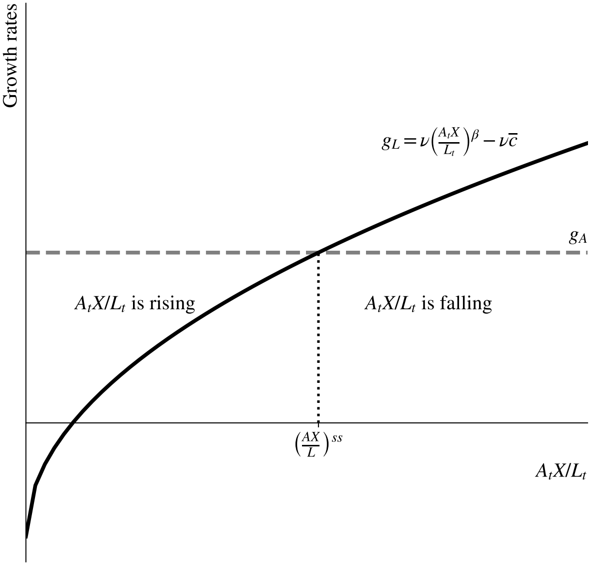


Plug in what we know about $y_t$ and you have

$$g_L = \nu \left( \frac{A_t X}{L_t} \right)^\beta - \nu \bar{c}. \tag{2}$$

- This is a dynamic system telling us that $g_L$ relates to a ratio $AX/L$.
- $g_L$ is negatively related to $L$. More people, lower living standards, lower population growth.
- This is similar analysis to Solow/Romer and other dynamic models.

# Malthusian dynamics



# Malthusian steady state



In the steady state it's the case that $g_L^{ss} = g_A$. From production function we have

$$g_y = \beta(g_A - g_L). \tag{3}$$

so in steady state it must be that $g_y^{ss} = 0$. In the Malthusian world living standards don't grow. If $g_y = 0$, the it must be the

case that

$$y^{ss} = \frac{g_A}{\nu} + \bar{c}. \tag{4}$$

as this ensures $g_L = g_A$.

# Malthusian steady state

Given

$$y^{ss} = \frac{g_A}{\nu} + \bar{c}. \tag{5}$$

- Living standards are *higher* than the subsistence level $\bar{c}$
- How much higher depends on $g_A$. Productivity growth allows you to stay ahead.
- $\bar{c}$ isn't a biological minimum, it depends on culture/society as much as biology
- Malthusian economies can be relatively well-off, but stagnant

# Malthusian effects



Malthus is kind of depressing. A substantial loss of population:

- Raises living standards for the remaining people
- Who then start to have more children in response
- Which lowers living standards
- Until living standards are back at the level before the shock to population

This happened historically with the Black Death.

# Escaping Malthus

The world economy does not appear to be in a Malthusian situation, we have sustained economic growth. Two important elements to escape:

- Innovation/technology accelerated
- Population growth changed it's relationship to living standards

# Endogenize innovation

Our general structure was

$$g_A = \theta \frac{(s_R L_t)^\lambda}{A_t^{1-\phi}}, \tag{6}$$

but here let

- $s_R = 1$, or everyone could potentially innovate
- $\lambda = 1$
- $\phi = 1$. We know this is wrong in modern world, but could be applicable before that

which gives us

$$g_A = \theta L_t \tag{7}$$

# Endogenize innovation



Make one additional assumption that economy is always "close" to Malthusian equilibrium so that

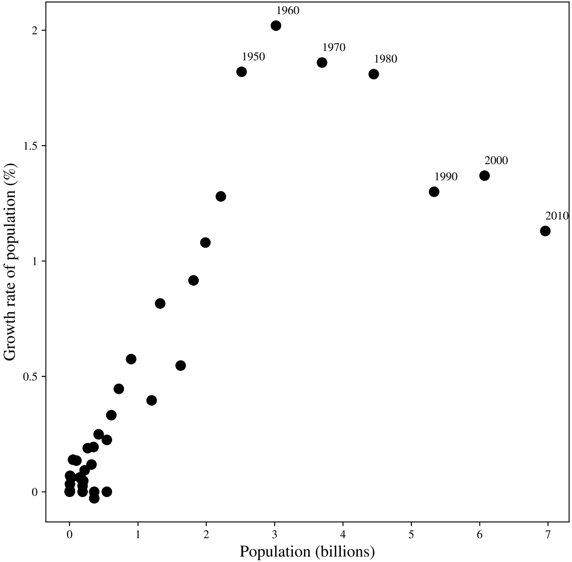
$$g_L \approx g_A \tag{8}$$

and then with endogenous innovation it would be that

$$g_L \approx \theta L_t \tag{9}$$

or population growth should rise with population size. As scale goes up, more innovation occurs, which raises living standards, which raises population growth, so scale goes up, ....

# From 1,000,000 BCE to the present



# Escaping Malthus



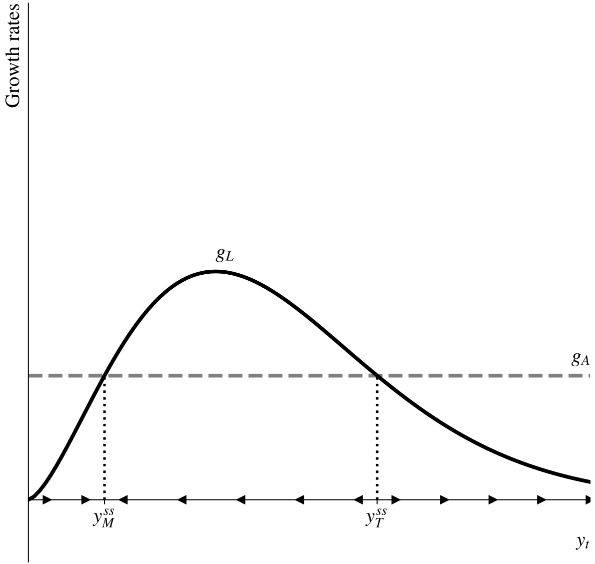


Endogenous technology means that $g_A$ goes up as $L$ goes up. By itself that cannot end Malthusian trap.

- $g_A$ keeps raising living standards, yes
- But population growth keeps growing
- Cannot break the stagnation problem
- And population growth cannot, biologically, continually get higher

What does a more realistic situation look like?

# Realistic function for $g_L$



# Escaping Malthus

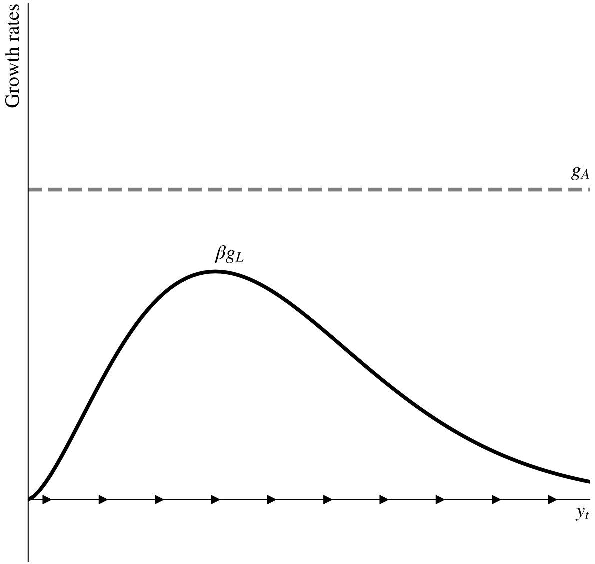




With realistic population growth function

- There is a Malthusian steady state at $y_M^{ss}$.
- If $y(0) < y_T^{ss}$ to begin, will end up in Malthusian state
- But if $y_t > Y_T^{ss}$, end up with sustained growth as $g_A > g_L$ always
- How did we get past this turning point?

# The transition to sustained growth



# Escaping Malthus



A reasonable story for the transition to sustained growth:

- The world/economy was in/near Malthusian steady state $y_{ss}^M$
- But at this steady state $g_L > 0$, so population grew
- Because $L$ grew, from endogenous innovation $g_A$ grew
- The level of $y_{ss}^M$ grew, so higher $g_L$, etc..
- And eventually $g_A$ was high enough that population growth could not keep up
- Which allowed growth to continue past the point of $y_{ss}^T$
- And entered the world where population growth *falls* with living standards
- Which puts us in the world of Solow/Romer/Schumpeter

# Early and late escapees



Some areas escaped Malthus before others:

- England is typical example of first industrializing nation around late 1700s (maybe earlier)
- But England and Europe were typically far poorer than China or much of Asia historically
- What makes sense for earlier take-off in Europe versus Asia?

# Using the Malthusian model



The *growth rate* of technology is more important than the *level* of technology:

- The escape from Malthus happens when $g_A > g_L$ for a sustained period of time
- Asia had large populations, so $g_A$ could be large
- But Europe may have had advantage in lower $g_L$ at any given level of living standards?
- Or a fortunate burst of innovation, raising $g_A$ even for a few decades, was sufficient to get over the hump

# Family choice problem

Population growth is a choice, constrained by resources to have and keep kids alive. Let

$$U = c^{\gamma} n^{1-\gamma}.$$

and families care about consumption, $c$, and number of kids $n$. $\gamma$ tells us how much they care about each. Their budget is

$$y = c + p_n n. \tag{10}$$

and $p_n$ is the "cost" of a child in terms of time, resources, food, etc. There is a trade-off with consumption.

# Utility maximization



Standard conditions are

$$\begin{aligned} MU_c &= \gamma \frac{U}{c} \\ MU_n &= (1-\gamma)\frac{U}{n}. \end{aligned}$$

and

$$\frac{MU_n}{MU_c} = \frac{p_n}{1},$$

which can be solved with budget for

$$n = \frac{(1-\gamma)y}{p_n}. \tag{11}$$

Kids/population growth depends positively on income and negatively on their relative cost.

# The cost of kids



Let the cost of children be

$$p_n = \bar{c}e^{\eta y}. \tag{12}$$

- There is some subsistence cost, $\bar{c}$
- Their cost goes up with income, $y$
- Because of the $e^{\eta y}$ the cost is "convex" or increases faster as $y$ goes up
- This captures that as incomes go up, taking time for kids is more costly (people delay familiy formation)
- It also captures that as $y$ goes up you might invest more in kids (school, health) so having more kids gets even more expensive (send 2 kids through college rather than 4 through high school).

# Population growth



Put this together and you have

$$n = \frac{(1-\gamma)y}{\overline{c}e^{\eta y}}.$$

- Population growth depends in two ways on living standards, $y$
- $n$ goes up because of $y$ because families have more resources
- $n$ goes down with $y$ because the price of children rises

# Population growth



The two effects of $y$ change in how strong they are, leading to

$$\begin{aligned}
\frac{\partial g_L}{\partial y} &> 0 \text{ if } y < \frac{1}{\eta} \\
\frac{\partial g_L}{\partial y} &= 0 \text{ if } y = \frac{1}{\eta} \\
\frac{\partial g_L}{\partial y} &< 0 \text{ if } y > \frac{1}{\eta}.
\end{aligned}$$

At low levels of $y$, higher $y$ raises population growth. At high levels, higher $y$ lowers population growth. This creates the "hump" shape that allows for sustained growth.