

Nested production structures

Due Feb 3rd

For this assignment, you have to come up with a derivation of the aggregate markup for an economy that has many small producers with individual markups. There are N total individual producers. They each have a production function of $Y_i = A_i K_i^\alpha L_i^{1-\alpha}$, and the α terms are the same for all producers. Each producer charges a markup of μ_i of price over their marginal cost. You don't know (or care in this case) how or why they have the market power to do this, but the μ_i are unique to each producer. All firms take the wage (w) and rental rate (R) of capital as given, and there are a total of L workers and K units of capital in the entire economy.

Let the aggregate markup, M , be defined

$$M = \frac{PY}{wL + RK}$$

which just gives the ratio of nominal revenues (GDP) to nominal costs. An aggregate markup of 1 thus means that all GDP is accounted for by input costs, and there are no aggregate economic rents. Any aggregate markup over 1 indicates the presence of economic rents to some firms.

For this homework:

1. Show that the aggregate markup, M , is equal to the weighted average of all individual firm-level markups, with a weight equal to the firm's share in costs. That is, the weight for firm i is $(wL_i + RK_i)/(wL + RK)$.
2. Show that the aggregate markup, M , can also be written as the weighted *harmonic* average of all individual firm-level markups, where the weights are equal to sales. That is, the weight for firm i is $p_i y_i / PY$.

Your answer must be written in Latex. I won't accept hand-written versions. If you are not familiar with Latex, now is a great time to learn it. You can get a sample document [here](#) which contains basic instructions. Otherwise, introductions to Latex are available on Google, or upper level graduate students can offer some assistance.