

Nested production structures

Due Jan 27th

For this assignment, you need to write down a model that has the following features.

1. There are J top level industries (i.e. agriculture or services)
2. Within each top level industry j , there are K_j sub-industries (i.e. textiles or footwear under manufacturing)
3. Within each sub-industry jk , there are N_{jk} firms, who are all monopolistic competitors with one another.

The preferences of individuals are simple. They consume a top level final good, and they prefer to consume more of it. The top level final good is provided by a (competitive) final goods firm that combines the output of the J top-level industries in a CES production function with an elasticity of substitution of γ . There is also a preference/production weight of ϕ_j on each industry.

Each separate industry j has a (competitive) final goods firm that combines the output of the K_j sub-industries below it in a CES production function with an elasticity of substitution θ_j , and then sells this final good to consumers. Each sub-industry has a (competitive) final goods firm that combines the output of the N_{jk} firms below it in a CES production function with an elasticity of substitution σ_{jk} , and then sells this final good to the industry level final goods firm.

Each firm i in sub-industry jk has a production function of $Y_{ijk} = A_{ijk}X_{ijk}$, where A_{ijk} is their productivity and X_{ijk} is the amount of input that they use. Each of these firms is a monopolist in producing their specific good. There is a total stock of X of the input, and it is mobile across firms, sub-industries, and industries. All firms are price-takers in the market for the input, which earns a wage w .

The consumers are the providers of the input, and Xw is their total income, which they spend on the various industry goods.

You should provide a solution for the following items

1. The input share of any firm ijk in their sub-industry jk , so X_{ijk}/X_{jk} .
2. The input share of any sub-industry in their industry, so X_{jk}/X_j .
3. The input share of any industry in total inputs, so X_j/X .
4. The input share of any firm in total inputs, so X_{ijk}/X .
5. An expression for aggregate output, meaning the production of the top level final good aggregator that sells to consumers

Given those solutions, you should provide a discussion of the following:

1. Under what conditions will an increase in firm level productivity, A_{ijk} , would result in a larger expenditure share and input share for that firm.

2. Under what conditions will an increase in the average productivity of a sub-industry, call it \bar{A}_{jk} , result in a larger expenditure share and input share for each firm in that sub-industry?
3. Under what conditions will an increase in the average productivity of an industry, call it \bar{A}_j , result in a larger expenditure share and input share for each sub-industry in that industry?

Your answer must be written in Latex. I won't accept hand-written versions. If you are not familiar with Latex, now is a great time to learn it. You can get a sample document [here](#) which contains basic instructions. Otherwise, introductions to Latex are available on Google, or upper level graduate students can offer some assistance.