# Econ 7343 Midterm 1 Answers

## Problem 1

1. Consider a standard Solow model. The economy is in steady state to begin with at time zero. At time period $\hat{T}$ the value of $\epsilon_K$ goes *up* (but stays below one) and the change is permanent.

   a. Draw a diagram showing how $g_{K/Y}$ and $K/Y$ are related, and how the dynamics of the capital/output ratio react to this shock.

   b. Draw a diagram showing how the log of GDP per capita evolves over time.

   c. Draw a diagram showing how the growth rate of GDP per capita evolves over time.

   d. Draw a diagram showing how the rate of return on capital evolves over time.

   e. Does anything change if $\epsilon_K$ goes up to equal one?

## Answer 1

This is a problem with a parameter change that shifts the BGP, and we need to figure out how it gets there. It's a little like a problem where $A_0$ changes in that the BGP will shift and there will be a jump.

For the diagrams we'll draw those in class, but we can talk through what happens in math. First, if $\epsilon_K$ goes up we know several things about what happens in the long run.

a. The steady state level of $K/Y$ does *not* change. That's still $s_I/(\delta + g_A + g_L)$.

b. The growth rate of GDP per capita along the BGP does not change. That's still $g_A$

c. The *level* of the BGP does go *up*, because that is $\ln y^B GP = \epsilon_K/\epsilon_L \ln K/Y^{BGP} + \ln A_0 + g_A t$. Since $\epsilon_L = 1 - \epsilon_K$, that means the leading term on capital/output goes up. So we know that we have to end up at a new, higher, BGP.

d. The steady state of the rate of return does not change, because the steady state level of $K/Y$ does not change, and we did not change $s_K$. If you made the additional assumption that $s_K \approx \epsilon_K$, then you might think that the rate of return on capital did shift up, but you'd have to make that assumption clear.

What we need to know is what happens immediately at $\hat{T}$. Here's where things get a little weird. Does this jump in $\epsilon_K$ change $K/Y$ (and hence $R$ and $\ln y$ immediately)? The answer is yes. There are a few ways to see this. Kind of formally, note that $d\ln K/Y = d\ln K - \epsilon_K d\ln K - \epsilon_L(d\ln A + d\ln L)$ or $d\ln K/Y = \epsilon_L(d\ln K - d\ln A - d\ln L)$. If you integrate this you get that $\ln K/Y = \epsilon_L \ln K/AL$.

When $\epsilon_K$ goes up, $\epsilon_L$ goes down. K, A, and L are all the same as they were before. Whether $K/Y$ goes up or down depends on whether $\ln K/AL$ is positive or negative, and that depends on whether $K > AL$ or not, which is weird. But we know $K/Y > 1$ in the data, at least as far as we measured it, so that implies that $K/AL > 1$. So it should be that when $\epsilon_K$ goes up, $K/Y$ goes *down*. The idea is that you made capital more important to production, and that immediately raised $Y$ in response (because A and L are relatively small), so $K/Y$ goes down.

That means two things for us. First, it means that with $K/Y$ dropping, the growth rate of $g_{K/Y} > 0$, and hence $g_y > g_A$ immediately. Second, it means that $\ln y$ jumps up too, but not all the way to the new BGP. So the economy gets an immediate boost, and then the growth rate is high for a while as the economy moves to the new BGP.

The last part of the question asked about if $\epsilon_K = 1$, and yes, that gets even weirder. Because if $\epsilon_K$ goes to one, then there is no BGP. The leading term here is $1/0$ and hence goes to infinity. It's not that you get infinitely wealthy, but the model breaks down if this happens because now $d \ln Y = d \ln K$ and that's it. $g_y = g_K - g_L$, and so $g_y = s_I Y/K - \delta - g_L$. But because there are no diminishing returns to capital the Y/K ratio is always constant at wherever it starts. So $g_y$ is just fixed by the initial $K/Y$ ratio always and forever.

## Problem 2

2. Compare two economies that have identical parameters and initial conditions for capital, labor, and productivity. The only exception is that in one economy $\epsilon_K$ is "big" (but below one) and in the other $\epsilon_K$ is "small" (but above zero). It will be useful to use your diagram from problem 1(a) to think about this. Both economies start out in steady state. Imagine that both economies are hit by an identical shock to the stock of capital, and that it falls by 10%.

   a. In response to this shock, for which economy is the drop in GDP per capita larger? Explain.

   b. In response to this shock, for which economy does it take longer to get back to steady state? Explain.

   c. Draw a diagram of log GDP per capita over time, and show the BGP for both economies, as well as their path of GDP per capita in response to the shock.

## Answer 2

Ok, this is similar to the first problem, of course. But now we're not thinking about how they act if $\epsilon_K$ changes, but comparing two countries, one of which is "capital intense" and one which is not. The figure relating $g_{K/Y}$ and $K/Y$ is key here.

For a 10% drop in capital, the economy with $\epsilon_K$ "big" has the bigger drop in GDP per capita. Capital matters more, so they feel the effects of this more. If you aren't sure about that, note that from $d \ln Y = \epsilon_K d \ln K + \epsilon_L (d \ln L + d \ln A)$ that should make clear that if $\ln K$ changes by the same amount (drops by 0.10) then the shock to GDP is bigger the larger is $\epsilon_K$.

However, the drop in K/Y is smaller when $\epsilon_K$ is big. $d \ln K/Y = \epsilon_L d \ln K - \epsilon_L (d \ln L + d \ln A)$, so when $\epsilon_K$ is big the shock to K isn't as big a deal to K/Y, mainly because Y adjusts to it a lot.

Now, the more interesting part of this is what happens in response. Both drop in terms of $K/Y$, and then have to go back to steady state. The growth rate of $g_{K/Y} = \epsilon_L (s_I / K/Y - \delta - g_A - g_L)$. So for the country with big $\epsilon_K$ the growth rate at any level of $K/Y$ is *lower*. That is, the curve in the K/Y diagram is flatter, and hence it moves more slowly towards steady state. The big $\epsilon_K$ country gets a smaller shock to K/Y, but responds to it more slowly.

So for GDP per capita the big $\epsilon_K$ country has a smaller drop, but then comes back more slowly to the BGP.

## Problem 3

3. Consider a standard Solow model, but the capital accumulation process is different. Rather than capital depreciating as it gets used, the capital stock *grows* as it gets used. One way to think about this is that as we accumulate capital, we aren't sure how it works, but as we use it more we find better ways to use it. Regardless, let $g_K = s_I Y/K + \delta$, where the important difference is that the rate $\delta$ is now added rather than subtracted.

   a. Under what conditions will an economy that works this way still have a steady state? What is the steady state value of $K/Y$?

   b. Assume the size of $\delta$ is such that there is no steady state for $K/Y$. What's the long-run growth rate of GDP per capita?

## Answer 3

Okay, so this just tweaks the basic model to essentially build in some growth in capital even if $Y/K$ gets close to zero. What this means is that $g_{K/Y} = \epsilon_L(s_I/K/Y + \delta - g_A - g_L)$. Does this have a steady state? Well, it retains the property that as $K/Y$ goes up $g_{K/Y}$ goes down, so it's possible it has a steady state. When K/Y is close to zero, the growth rate is close to infinity. What happens as $K/Y$ gets very big? The leading term goes to zero, and $g_{K/Y} \to \epsilon_L(\delta - g_A - g_L)$. This is only negative if $g_A + g_L > \delta$. If that holds, then at a high enough K/Y ratio $g_{K/Y} < 0$, and therefore $g_{K/Y} = 0$ at some point, or there is a steady state. At that steady state, $K/Y^* = s_I/(g_A + g_L - \delta)$.

If $\delta$ is big enough, then there is no steady state and $K/Y$ keeps growing forever, but what is that rate? Well, as $K/Y$ gets very big $g_{K/Y} \to \epsilon_L(\delta - g_A - g_L)$, so the right-hand side is the growth rate of the capital output ratio in the long-run. That means $g_y = \epsilon_K(\delta - g_A - g_L) + g_A$.

## Problem 4

4. For this question you need to refer to the figure showing the time path of the rate of return $R$ for two countries, A and B. Assume that both have the same steady state rate prior to the shock in period 10. Prior to the shock they have the same level of GDP per capita. You can assume that B returns to the same rate as it started with.

   a. Describe what possible shocks could have caused the path for the rate of return in country $A$.

   b. Describe what possible shocks could have caused the path for the rate of return in country $B$.

   c. Now, I give you the additional information that at the end of the time shown, country $A$ has a higher GDP per capita than country $B$. Does that allow you to narrow down what happened in either or both countries?

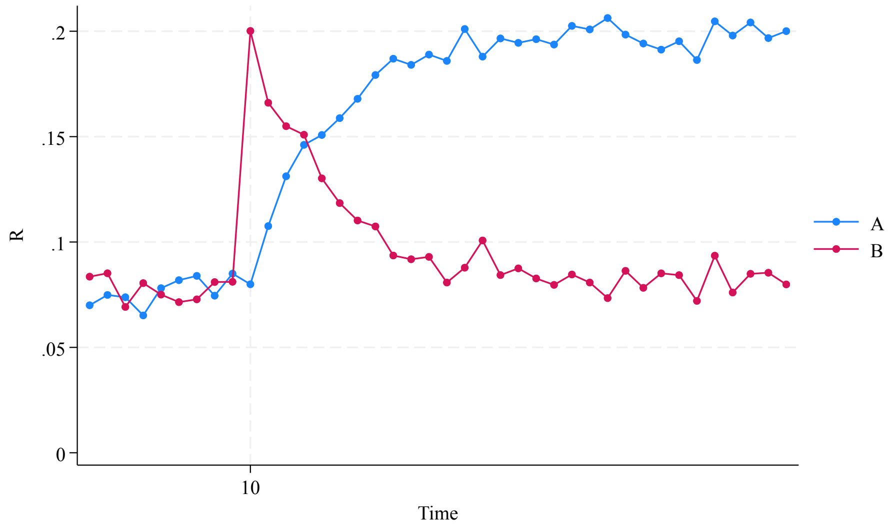


Figure: Two countries

d. Now, ignore part (c). I give you the information that at the end of the time shown, country $A$ has a higher consumption per capita than country $B$. Does that allow you to narrow down what happened in either or both countries?

## Answer 4

For A, nothing happens to R immediately, but it moves to a higher steady state return to capital. We know the return in steady state is $R = s_K(\delta + g_A + g_L)/s_I$, so it has to be one or more of these parameters that changed to cause that shift upwards. At this point, it's hard to say anything other than one or more of them changed.

For B, there was a distinct jump and then return to steady state, so this looks more like a shock to K (fell), A (rose), or L (rose). This seems like K/Y went down, and then thanks to the dynamics it recovered back to steady state. But without more information it's hard to pin that down.

So for part c, now there is a little more information. Country A ends up richer than country B. If that's true, and A has higher GDP per capita than B, then we know that after this change it must be that the parameter that raised R in country A also raised the level of the BGP. That eliminates $s_I$ going up, because that would have lowered the long-run rate of return. It eliminates $s_K$, because that doesn't influence the level of the BGP at all. It eliminates $g_L$ or $\delta$ going up, because those would have lowered the BGP. The only real answer here is that it must be that $g_A$ went up in country A to deliver a higher long-run return to capital *and* a higher GDP per capita.

Okay, for d start over. Now we know that consumption is higher in A than in B by the end of the series. I'm working again on the assumption they start with the same, and again if you weren't sure on this that's fine. Now, one answer could be that $g_A$ went up in A. That would lead to both a higher return, a higher BGP for GDP per capita, and a higher amount of consumption per capita. But it could *also* have been a decline in $s_I$ that raised the rate of return but in this case *raised* consumption per capita. That only works if you assert that it must have been that $s_I > \epsilon_K$ - above the Golden Rule - to begin with.

## Problem 5

5. Answer the following short answer questions:

   a. What are the four characteristics of a balanced growth path?

   b. In a standard Solow model, what's the steady state value of $K/Y$?

   c. What ensures that the steady state in a Solow model is stable?

   d. What's the difference between $s_K$ and $\epsilon_K$?

   e. Why does $g_L$ only affect the level of the balanced growth path, and not the growth rate on the BGP?

## Answer 5

a. Apologies because the definition in the text wasn't exactly four points (bouncing between definitions). Constant or no trend in the growth rate of GDP per capita, consumption, and capital. Constant capital/output ratio. Constant share $s_I$ and constant share $s_K$ would be valid too. Terrible question.

b. $s_I/(\delta + g_A + g_L)$

c. That the growth rate of capital/output goes down as K/Y goes up, and the precise answer would include that the growth rate is positive when K/Y is small and negative when K/Y is big.

d. Share of GDP paid to capital versus elasticity of GDP with respect to capital

e. Because the growth rate of capital adjusts in response to the growth rate of GDP (which depends on $g_L$) to ensure a steady state in $K/Y$, essentially. An impact of $g_L$ on the growth rate is temporary as eventually capital adjusts.