Econ 7343 Midterm 2 Answers

Problem 1 and 2

1. Consider a standard neo-classical economy with individual consumers, firms, and a zero-profit financial system. This economy has a "small" intertemporal elasticity of substitution. There is positive productivity growth. The economy starts at the steady state k/y ratio and is on a balanced growth path. This economy experiences a negative shock to the level of productivity, meaning A_0 goes down, while the trend rate of productivity growth, g_A, stays the same.

    a. Draw a figure showing how (log) GDP per capita, $\ln y$ evolves over time in response to this shock.

    b. Draw a figure showing how the growth rate of GDP per capita, g_y, evolves over time in response to this shock.

    c. Draw a figure showing how (log) consumption per capita, $\ln c$, evolves over time in response to this shock.

    d. Draw a figure showing how (log) capital per capita $\ln k$, evolves over time in response to this shock.

    e. Draw a figure showing how the rate of return on capital, r, evolves over time in response to this shock.

    f. Draw a figure showing how the savings rates, s_I, evolves over time in response to this shock (and note any assumptions you had to make to draw this figure).

2. Consider a *different* economy that has a "big" intertemporal elasticity of substitution, and which has parameters such that it has an identical steady state level of $(k/y)^*$ and an identical balanced growth path for GDP per capita and consumption as the economy in problem 1. This economy is hit by an identical shock to productivity as the economy in problem 1.

    a. For each of the figures in Problem 1, draw in the path of each variable for this economy with a "big" intertemporal elasticity of substitition, making clear how it differs from the economy in Problem 1.

    b. For the savings rate, explain how or why the two economies might have different responses. I'm looking for intuition here, not necesarily math.

Answer 1 and 2

1. These are the same problem, just asking you to show the difference in how the IES impacts the response to the shock. So you needed to identify what a negative productivity shock does to all these variables, and then show how 1/sigma dictates the different responses.

    a. The negative shock to productivity means that $\ln y$ has a distinct drop down immediately, and then drops further over time as it reaches the new, lower BGP in both economies. The immediate drop is the same in both economies. The transition to the new BGP goes faster with the big IES, and slower with the small IES.

b. There is an immediate big drop in the growth rate, which is what matches the distinct drop in $\ln y$. From that point the growth rate is lower than g_A and rises to g_A over time. It rises faster with a high IES and slower with a low IES. Why does the growth rate stay below g_A? Because the drop in A_0 means that k/y goes *up* immediately after the shock, which lower the growth rate $g_{ky} < 0$. When things are less productive, it looks to the individuals like they have too much capital.

c. Consumption has a distinct drop immediately, as with GDP per capita. It then also falls further to a new, lower BGP for consumption. Here's where the IES matters. For people with the small IES the immediate drop in consumption is bigger, but then they take longer to get back to the BGP. Big IES people have a smaller drop but then rapid decline in consumption to the new BGP.

d. Nothing happens immediately to k. But then it begins to decline to a new, lower BGP. Same idea with IES. Slow transition with low IES. Fast transition with high IES.

e. The rate of return on capital drops immediately, because of the increase in k/y. This is also what induces people to slow their consumption growth. It then slowly transitions back to the original steady state, because nothing changed that long-run value. Slower with low IES, faster with high IES.

f. Whatever the immediate response of the savings rate, we know that in the long run it goes back to the same steady state, as none of those parameters changed. Whether savings jumps or falls in response is really a question of how big $1/\sigma$ is relative to s_I^* to begin with, and so there is not "right" answer to which direction it should jump. *But*, we know that the high IES economy's savings rate will react by more, and then transition back to the steady state s_I^* faster. The low IES economy's savings rate will only respond a little and then take longer to get back to s_I^{ast}.

Problem 3

3. Consider a forward looking consumer with infinite time periods and typical lifetime utility. They face a constant interest rate of r. They have zero initial assets ($a_0 = 0$). They have income of w_t each period, and that income grows at the rate g such that $w_{t+1} = (1+g)w_t$.

   a. What is the Euler equation for this person?

   b. What is their initial consumption, c_0?

   c. Under what conditions on r and g will this person have $c_0 < w_0$, meaning they save some of their initial income?

   d. This person is consuming according to their optimal plan based on the given information. In period J, they experience an unexpected shock to the wage process so that the entire path of wages from time J forward is lower by some fixed percent (e.g. they have to take a pay cut of 10% that holds in period J and for every period thereafter). Draw a figure showing how their consumption evolves over time in response to the shock (make sure to show how consumption was evolving before the shock, too).

e. Now, imagine that this person learns about the negative shock in period J at some prior period (say, two years before). On the same figure as (d), show how their consumption responds to the information arriving, and show how or if their consumption changes when the shock actually occurs.

f. Without having to draw anything, explain how the response of this person to the shock would differ if the conditions were such that $c_0 > w_0$ in the initial period, versus if $c_0 < w_0$ in the initial period.

Answer 3

1. Just a consumption problem with some assumptions about the path of wages.

   a. Typical. $U'(c_t)/U'(c_{t+1}) = \beta(1+r)$ but other notations (using g_c or θ) are fine too

   b. The problem didn't say, so presumably they have CRRA utility, otherwise you wouldn't be able to solve this explicitly. But with CRRA we know that $1 + g_c = (\beta(1+r))^{1/\sigma}$. Consumption in any period t is $c_t = c_0(1+g_c)^t$. So lifetime budget constraint is $\sum_0^\infty c_0(1+g_c)^t/(1+r)^t = \sum_0^\infty w_0(1+g)^t/(1+r)^t$. That solves for $c_0 = w_0(r-g_c)/(r-g)$.

   c. $c_0 < w_0$ if $r - g_c < r - g$, or if $g < g_c$, or if $1 + g < (\beta(1+r))^{1/\sigma}$, or if $(1+g)^\sigma/\beta < 1 + r$. That is, if r is *high* enough or g is low enough, this person will save some of their initial income and use that to build up consumption later in life.

   d. This problem basically is like solving a brand new c_0 problem at time period J. Because lifetime wealth is now lower than before, this person is going to drop their consumption in period J by some amount, and shift down to a parallel path for consumption from J to infinty. Nothing changed about r or g at the shock, so the growth rate of consumption must be the same after the shock.

   e. The key here is that they'll respond when they *learn* about the shock (two years before) and adjust their consumption a little, but then there will not be any shift in consumption when the shock actually arrives, because they'll have taken this into account already.

   f. The difference in the two situations is that someone who started with $c_0 > w_0$ was borrowing against future income, and now that future income just got cut, so they will have to drop consumption a lot in response to the shock, because in period J they start with lower initial assets a_J (could be negative if they are still borrowing, or could just be a small savings). Someone with $c_0 < w_0$ was building up savings already, so they have bigger a_J at the time of the shock. They won't need to drop as much.

Problem 4

4. For this question you need to refer to the figure showing the time path of the savings rate s_I for two countries, A and B. Assume that both have the same steady state rate rate prior to the shock in period 10. Prior to the shock they have the same level of GDP per capita. You can assume that both A and

B are moving to the same steady state rate of $s_I \approx 0.15$ after the shock. You can assume that they are hit by the same shock, whatever it is.

a. Describe what possible shocks could have caused the path for s_I in both countries.

b. Describe what possible reasons there are that the paths for A and B look different, and what you can conclude about how any parameter(s) might differ between A and B.

c. Now, I give you the additional information that at the end of the time shown, both countries have the same rate of return r as they did before the shock. Does that allow you to be more specific about what shock hit the two countries?

d. In period 20, which country has the higher level of consumption? Explain how or if your answer depends on the kind of shock that happened, and how or if there is not enough information to tell.

Answer 4

1. This is a hard question, so consider this an example of about the worst it could get on a comp. There's a lot to keep track of to see what could/did occur.

   a. Both economies had savings drop at the time of the shock, and then are transitioning to a lower steady state s_I^*. Because of that change in steady state we know this wasn't a one-off shock to K, L, or A, none of which would have shifted the steady state savings rate. It must be some combination of higher θ, lower ϵ_K, lower g_L, or some weird change to δ or g_A that shifted s_I down. There isn't really anything here to distinguish which of those was responsible.

   b. A and B must differ in their IES (1/sigma). A responds in a "bigger" way to the shock, lowering the savings rate by a lot, and then transitions back to the steady state relatively quickly. This would be consistent with having a higher IES (lower σ). B has a smaller reponse and then a longer transition, again consistent with a low IES (and high σ).

   c. Both countries end up with the same r^*, which means it cannot be a change in θ or g_A that induced the change in s_I^*, because otherwise that would have changed the long-run return to capital. We're left with either ϵ_K or g_L. Yes, it could also be a change in δ of just the right kind, but I did not think through that writing the question (I gave you credit for any explanations relying on δ). Can we distinguish between the ϵ_K or g_L shocks? Yes. If ϵ_K fell (lowering s_I^*), then that would also imply a lower BGP for GDP per capita and consumption, both A and B will feel poorer and need to transition *down* to a lower path for consumption. To avoid the massive shock to consumption they both will *lower* their savings rate to some extent, and smooth out the transition. "A" lowers savings by a lot to keep consumption high early and then accepts a rapid decline in consumption to the new BGP. "B" lowers savings by a little, which means consumption drops by more right away, but then they have less change in consumption along their transition, consistent with their low IES. This all tracks with what we see in the Figure.

   d. Given the above, ambiguous is the answer. "A" starts with higher consumption and transitions down the new BGP faster, which means at some point their consumption must fall below "B". But we don't know when that will happen, so at period 20 it could be either way.

Why is the answer to Question 4 not "g_L went down?". On the other had, if g_L had gone down, then both countries would transition to a *higher* BGP for *consumption*, but their BGP for GDP per capita would not have changed, as their steady state k/y ratios would not have changed at all. They could just change their minds immediately to save less and consume more *and there would be no transition in savings.* There's nothing to transition, because they don't have to get GDP per capita to move to another BGP - it stays right where it is. Because savings dropped for both, it must be that they are transitioning to a new BGP for GDP per capita, which means it can only be ϵ_K that fell.