Econ 7343 Midterm 3

This is closed book and closed note. You should write your answers on blank paper. You should start each problem on a new sheet of paper. Put your name on *every* sheet of paper you turn in. You should read each question carefully before starting to answer.

Problem 1

1. Assume you have a Ramsey model with standard assumptions about preferences, capital, and production. There is uncertainty about the level of productivity each period. Productivity can either be A^{High} or A^{Low}. The Markov transition matrix is $\begin{bmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{bmatrix}$ where the columns are the probabilities of going into the High productivity state (1st column) and the Low productivity state (2nd column) next period. The first row are the probabilities if you start in the High productivity state, the second row is if you start in the Low productivity state. Write down two value functions associated with this model, and include the probabilities from the Markov transition matrix explicitly in your value functions.

Answer 1

We know we have a set of 2 Bellman equations because there are two states of the world. With everything standard, it should look like this:

$$v(k|Low) = \max_{k'} \left(u(y^{High} + (1 - \delta - g_L)k - k') + \beta(.8v(k'|High) + .2v(k'|Low)) \right)$$

and

$$v(k|High) = \max_{k'} \left(u(y^{Low} + (1 - \delta - g_L)k - k') + \beta(.1v(k'|High) + .9v(k'|Low)) \right)$$

The most important part is the probability weights on the value functions, and the details of the constraint inside $u(c)$ depend on your exact assumptions about how the economy works. But the key here was to identify that the expected future value function is a mix of the value of the High and Low values.

Problem 2

2. There is a process $c_t = \mu + \rho c_{t-1} + \varepsilon_t$, where ε_t is a stochastic shock, μ is a constant, and ρ is a constant.

   a. Write this as an infinite moving-average process.

   b. What are the conditions under which the process for c_t is stationary?

   c. Now, assume that c_t does not meet your conditions for (b). What are the conditions under which the process for c_t would be difference stationary?

Answer 2

2. Should be straightforward, the only tweak as the fixed value μ involved,

   a. $c_t = \sum_{j=0}^{\infty} \rho^j \mu + \sum_{j=0}^{\infty} \rho^j \varepsilon_{t-j}$

   b. So long as $|\rho| < 1$ this will be stationary, because the sum of shocks will be finite. That will also ensure that the summation over μ is just $\mu/(1-\rho)$, which is finite. Otherwise the expected value of this thing will be infinity.

   c. If $|\rho| \geq 1$ then this will not be stationary. To be general, in first differences this would be $c_t - c_{t-1} = (\rho - 1)c_{t-1} + \varepsilon_t$. This would be difference stationary for sure if $\rho = 1$. If ρ is any other value then this isn't necessarily difference stationary.

Problem 3

3. Think about a standard Ramsey problem with forward-looking consumers who have standard CRRA utility. Capital and production all work in the standard ways. Answer the following questions *without deriving any specific formulas.* The entire point of this question is to explain what is going on, not to write down equations. You can use some math as shorthand to help explain things (e.g. "$u'(c)$ would be higher if ...."), but I want *explanations* not just showing me an equation.

   a. Explain why people would do "precautionary savings", and be explicit about what this assumes about utility functions.

   b. Explain whether people who have a high intertemporal elasticity of substitution will do more or less precautionary saving than people with a low intertemporal elasticity of substitution, and why.

   c. Explain how the variance of expected shocks to productivity in the future influences the savings rate in the Ramsey model.

   d. All else equal, would an economy with a big intertemporal elasticity of substitution have a higher or lower level of expected consumption along the balanced growth path than an economy with a small intertemporal elasticity of substitution?

Answer 3

3. Mainly just understanding this concept. Be careful on how people with different IES have different preferences for precautionary saving.

   a. Precautionary savings is savings done to offset the effects of uncertainty, and is over and above the "normal" amount of savings done because of impatience or consumption smoothing motives. People dislike uncertainty, and to alleviate that they save more so that they feel richer in the future, and when the shock occurs it has less of an effect on their utility because they are richer. This is referred to as "prudence" in many cases, and it occurs so long as $u'''(c) > 0$. In practice, that condition on the third derivative means that while marginal utility declines with consumption ($u''(c) < 0$), it does so at a decreasing rate. That is, $u'''(c) > 0$ means that marginal utility

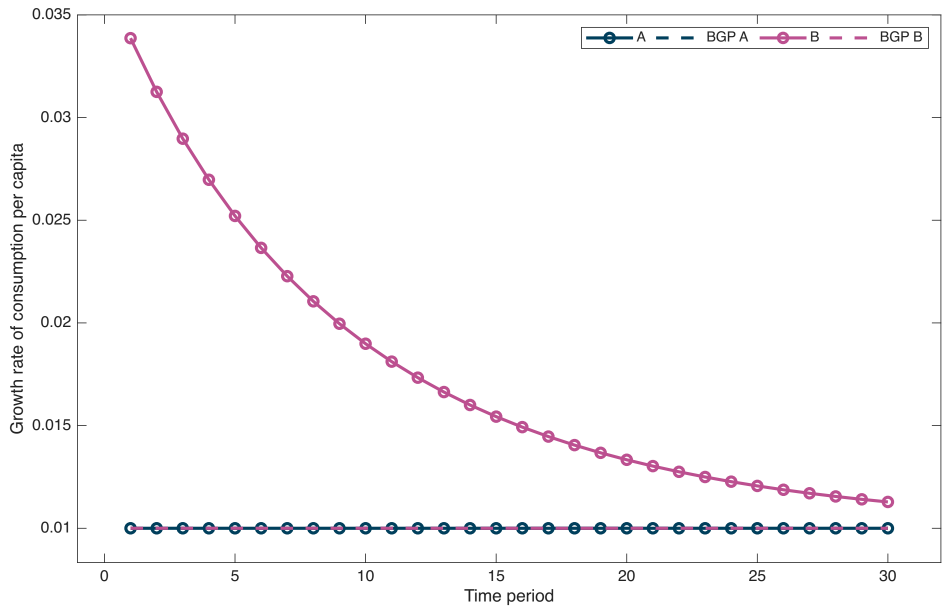

Figure: Response to shock in B

"flattens out" with respect to c once c is big enough, meaning it does not change with c by much at all. If marginal utility doesn't change with c, then when the shock hits the marginal utility of consumption is similar no matter whether the shock was good or bad, and so the person doesn't have to worry about it.

b. Less. High IES people are less worried about differences in consumption across time or states of the world. Their marginal utility doesn't change alot because c differs, so they are less worried about things that cause c to differ, so they are less willing to do precautionary savings because it doesn't matter to them as much. One way to see things is to say that $u'(c) = c^{-\sigma}$ with CRRA. Note that the elasticity of $u'(c)$ with respect to c is $-\sigma$. The lower is sigma (higher IES) the less elastic $u'(c)$ is with respect to c.

c. Because it increases the dispersion in c that the person will experience when the shock occurs, and therefore the dispersion in MU is bigger, and people dislike dispersion in their MU across situations. Hence, if the variance of the shock is big they will save more in order to mediate that dispersion.

d. Lower. For the same logic in (b), high IES economies are less worried about uncertainty, and so do less precautionary savings. Because they do less precautionary savings, they have a lower level of expected K/Y and a lower BGP for log GDP per capita.

Problem 4

4. This problem refers to the figure. What is plotted is the growth rate of consumption per capita, g_c. There is a baseline economy A where the growth rate is always 1%. Economy B started off on the

exact same baseline as economy A, with the same BGP and steady state values for all variables. But then some shock hit Economy B leading to the path you see here for the growth rate of consumption, in red. You can assume that the growth rate of consumption in B is headed back towards the same baseline rate of 1% as in Economy A.

a. What are the plausible shocks that could have hit Economy B to cause this jump in g_c? The goal here is to list all the plausible shocks, not to pick just one.

b. What does the data for g_c imply about the path of the savings rate in Economy B, if anything?

c. What does the data for g_c imply about the path of the rate of return in Economy B, if anything?

d. What does the data for g_c imply about the path of the growth rate of GDP per capita, g_y, in Economy B, if anything?

e. What does the data for g_c imply about the level of the BGP for GDP per capita in Economy B, if anything?

f. Now, I tell you that in the long run the rate of return, r^*, is *lower* in Economy B than in the baseline (Economy A). What can you infer about the plausible shocks that hit Economy B?

Answer 4

4. In my answer I mention that a change in the degree of uncertainty could be a reason, but that was not a focus in class, and if you didn't talk about that it was not penalized.

a. Lots of things. It could be lower θ. It could be more "noise" or uncertainty, in the crude way we talked about in class. It could be a higher ϵ_K. It could be a higher A shock. It could be higher L shock. It could be a lower K shock. We know it is *not* a shock to g_A because that would mean that in the long run it ended up with a different growth rate of consumption than Economy A. We know it was not g_L because that would have just jumped to the new consumption BGP and not involved a change in growth rates.

b. Not much. Depending on the size of σ and the specific shock that occurred, it's possible that the savings rate could have jumped up (if σ was relatively small) or dropped (if σ was relatively big). In addition, some of those shocks imply that the steady state savings rate would have shifted, which means we'd expect savings to jump to begin with and then move to some new steady state. We can infer very little.

c. We know a little. We don't know whether r jumped up in B to start with (if it was A, K, or L) or if r was the same in A and B to start with (parameter shocks). But we do know that wherever it started, the rate of return must have *fallen* over time back to steady state. Why? Because the only thing consistent with the path of consumption growth is that r went down over time. Think of the Euler equation.

d. It must look similar to the path for g_c. We konw it wasn't g_A that changed, so ultimately the growth rate of GDP per capita must have gone back to the same as in Economy A. All the shocks that could explain this would imply faster growth in GDP per capita at the outset. Either the

BGP shifted up (parameters) or the K or L shock put the economy below its BGP, or a shock to A shifted the BGP up (and caused a little jump in GDP per capita).

e. While we know what g_y must look like, we don't know what caused it, so we don't know what the level of the BGP looks like. It could have stayed the same (K, L), it could have shifted up (A, parameters).

f. Now we have some information. If r^* ends up *lower*, then it has to be that θ went down OR uncertainty went up. Those two things influence the long-run rate of return because they show up in the Euler equation which pins down r.

Problem 5

5. Think of a typical Ramsey/Neo-classical economy with optimizing consumers, firms, and a financial sector. The economy starts on a BGP, and then at some time J is hit by a permanent *decrease* in their discount rate, θ. Make sure to read all the parts of the question first before you start answering, so that you leave yourself room in the figures.

   a. Draw a figure showing the path of $\ln c_t$ over time, both before and after the shock, being sure to draw any BGP(s) and the actual path of the variable.

   b. Draw a figure showing the path of r_t over time, both before and after the shock, being sure to draw any BGP(s) and the actual path of the variable.

   c. Draw a figure showing the path of s_{It} over time, both before and after the shock, being sure to draw any BGP(s) and the actual path of the variable.

   d. Using the same figures from (a), (b), and (c), draw the paths for another economy that experiences the same exact shock to the discount rate, but which has a much *larger* value of σ than your original answer.

   e. Think about part (a). Explain why the BGP of consumption did, or did not, shift in response to the decrease in θ, and why that shift, if it occurred, was in the direction you specified.

Answer 5

5. People get more patient, and I wanted pictures of how two economies react depending on their σ values. Start here with part (e). We know that the BGP of consumption shifted *up*. Why? Because our Ramsey model is below the Golden Rule, and hence an increase in the savings rate, which θ will create, will increase consumption. So for both economies we know they are shifting to a higher BGP for both GDP per capita and consumption.

   a. Consumption drops *below* the old BGP, and then grows to reach the new higher BGP. It drops below because people get more patient and are immediately will to save more. The drop is bigger for the high IES (low sigma) economy, but it gets back to the new higher BGP faster.

   b. The rate of return falls over time in both because lower θ means lower r^*. It falls *faster* in the high IES (low sigma) economy.

c. The savings rate jumps in both, and they both head to the same long-run steady state savings rate. What we also know is that the high IES economy's savings rate will jump up *more* than the low IES one. Depending on your guess/assumption, it could be that the high IES savings rate jumps *above* the new steady state and then falls, and the low IES savings rate jumps to just *below* the new steady state and then rises. But both could jump above, or both could jump below. The important part is that the high IES economy jumps *higher*.

d. Explained in prior answers. It was a lot easier to answer this if you put the two economies on the *same* graphs.