Midterm Exam

Econ 7343, Macroeconomics I

There are 100 total points on the test. You have three hours to complete it. The test is closed book and closed note. You will need only something to write with, and blank paper. You should number the pages of your answers, so that I can keep track of them while grading. Make sure you read each question carefully, and answer all of the parts of the question.

1. [**20 points**] Consider the following figure showing the level (in logs) of GDP per capita in France over the very long term. As you can see, I've plotted using dashed lines what look like two different balanced growth paths for France. Note that after the 1970's, the growth rate (slope of the dashed line) is *higher* than it was before the 1950's. Using the basic structure of endogenous productivity growth, $\Delta E_{t+1}/E_t = \theta L_{Rt}^{\lambda}/E_t^{(1-\phi)}$, where L_{Rt} is the number of research workers, explain how it could be possible that the growth rate changed in France.

   [**Answer**]: One of the key things here is to keep the difference between a growth effect and a level effect. The question asks you about the *growth* effect seen in the figure, meaning the growth rate got higher. Within the structure of that accumulation equation for E, we know that the long-run growth rate is $\lambda n/(1-\phi)$, where n is the growth rate of L_R. So one possibility is that the growth rate of L_R went up. Another is that λ went up or ϕ went up. It is not true that a shock to E itself would raise the growth rate permanently, nor would an increase in the share of population doing research.

2. [**20 points**] Use the France diagram again. Now, ignore the fact that the growth rates along the two different balanced growth paths are different (assume they are the same).

   1. Within the context of a Ramsey model, explain (i.e. in words) what could have happened around the 1950's-1970's that would account for the shift from the lower BGP to the higher BGP.
   2. Draw a diagram of the time path of the return on capital (r_t) from 1800 to 2005 that matches your explanation for the shift in the BGP.
   3. Draw a diagram of the time path of capital per worker (k_t) from 1800 to 2005 that matches your explanation for the shift in the BGP.

   [**Answer**]: You have a few options here, which include an increase in the savings rate, a drop in the population growth rate, or a one-time shock to the level of E. Some people focused on the big dip in output per capita right before the shift up, which is just the end of World War II. But given that, one could think that perhaps it was a shift of people into research, s_R going up, which would have had that temporary drop in output per capita, and then shifted France up to a higher balance growth level. Let's assume for the other parts that it was a shift up in the savings rate (meaning an increase in beta). If that is true, then the rate of return on capital would have been approximately constant from 1800 to 1950, and then would have slowly declined to a lower long-run level, given the more patient people. For the capital per worker, this, would have been growing at a constant rate prior to 1950, and then had a temporary period of rapid growth until it ended up at a higher balanced growth path, again growing at a constant rate.

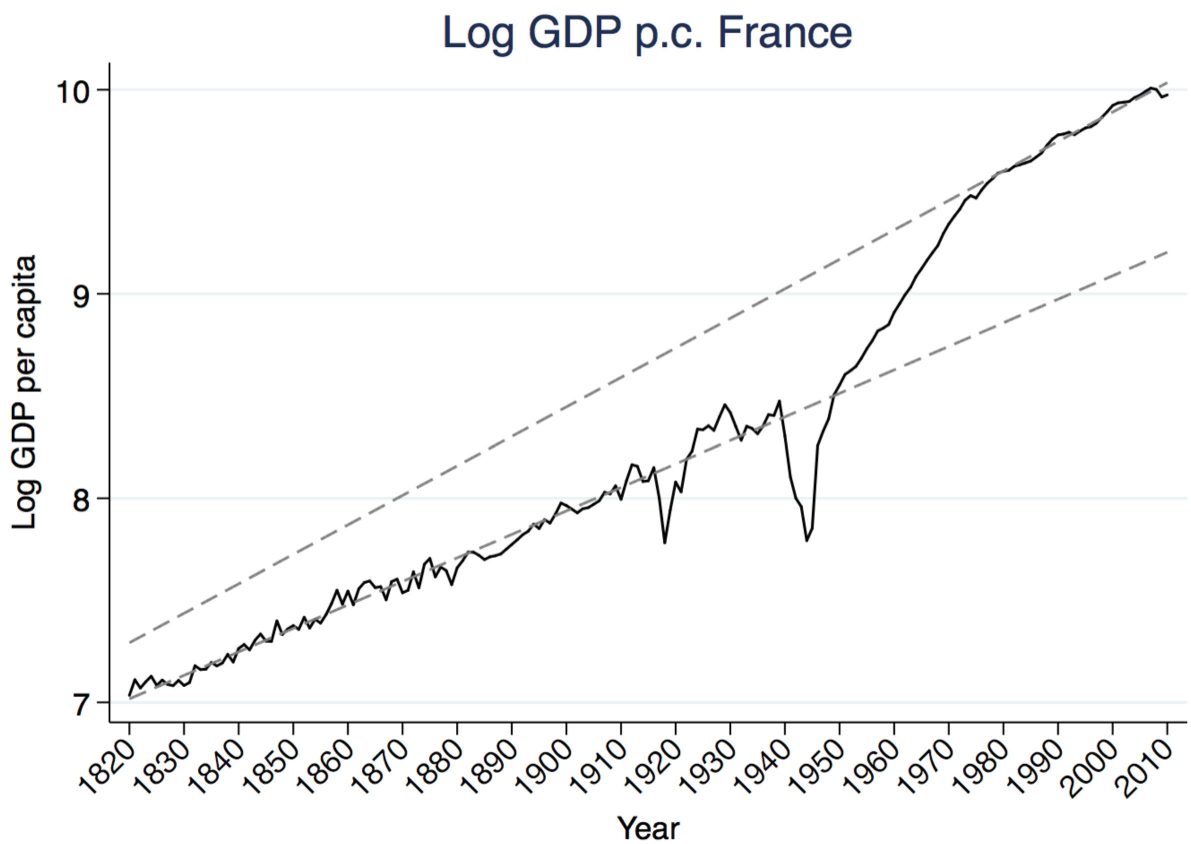

Figure 1: France GDP per capita

3. **[25 points]** Individuals are infinitely lived, and have standard forward looking lifetime utility with a time preference rate of β, and CRRA period utility with $c_t^(1-\sigma)/(1-\sigma)$. Production, in per worker terms, is $y_t = k_t^\alpha$. The payments to workers and capital owners, however, are not standard. Wages are $w_t = (1-\theta)k_t^\alpha$, and the gross return to capital is $R_t = \theta k_t^{\alpha-1}$, so the return on savings for individuals is $r_t = \theta k_t^{\alpha-1} - \delta$. There is no technological change or population growth.

   1. Write down the Lagrangian for these individuals.
   2. Write down the Euler equation for these individuals.
   3. Solve for the steady state level of capital per worker.
   4. The economy is in steady state. The value of θ is $\theta > \alpha$. There is s change in policy that then shifts $\theta = \alpha$. Draw a figure showing the time path of consumption per person over time, including what happens immediately at this shift, and then off into the future.
   5. For the same change in policy, draw a figure showing the time path of the return on capital, R_t, over time, including what happens immediately at this shift, and then off into the future.

   [Answer]: (1) The Lagrangian is $L = \sum[\beta^t U(c_t) + \lambda_t[(1+r_t)a_t + w_t - c_t - a_{t+1}]]$. These are individuals (not a central planner) so they do not appreciate that the rate of return depends on θ, they just take it as given. (2) The Euler equation is $U'(c_t)/U'(c_{t+1}) = \beta(1+r_t)$. (3) Now, given the Euler, we can plug in the return on capital, and find that $k^* = \left(\frac{\theta}{1/\beta - 1 + \delta}\right)^{1/(1-\alpha)}$. Notice that θ matters here. (4) With $\theta > \alpha$ it means that the s.s. capital stock is higher than it is after the shift. In other words, the $\Delta c/c$ line in the Ramsey diagram shifts to the left. With that, the initial reaction is for consumption to jump *up*, but then to decline over time to a lower steady state level of consumption. (5) At the same time, the rate of return on capital *drops* immediately, because we go from θ to α. But then it grows back to the original level, because the s.s. rate of return is still dictated by $1/\beta - 1 + \delta$, regardless of what the value of θ may be.

4. **[10 points]** You have an OLG economy, where individuals have utility of $V = \ln c_{1t} + \beta \ln c_{2t}$, and production takes place according to the increasing returns to scale function $y_t = k_t^\gamma$, where $\gamma > 1$. Labor earns a wage equal to $w_t = \theta k_t$, where $0 < \theta < 1$. There is no population growth. Show all the possible steady states of this model, and whether they are stable. Is it possible that output per worker (y_t) grows forever?

   [Answer]: This question has a typo. The wage should be $w_t = \theta k_t^\gamma$. If you used that, you get one unstable steady state at $k^* = \left(\frac{\beta}{1+\beta}\theta\right)^{1/(1-\gamma)}$. If $k_0 > k^*$, then this economy does have growth forever. If $k_0 < k^*$, then the economy ends up with zero capital. If you did the problem with $w_t = \theta k_t$, it devolves, because there is no clear way to get a steady state. In other words, the only possible steady state is k = 0. So you could get growth if $\theta\beta/(1+\beta) > 1$.

5. **[5 points]** The firm production function is $Y = [AK^{(\sigma-1)/\sigma} + BL^{(\sigma-1)/\sigma}]^{\sigma/(\sigma-1)}$. Assuming that that firms produce a homogenous good, and that they compete, and that they take the wage and rental price of capital as given, what is the share of output earned by capital, K?

   [Answer]: The share is $AK^{(\sigma-1)/\sigma}/[AK^{(\sigma-1)/\sigma} + BL^{(\sigma-1)/\sigma}]$.

6. **[5 points]** If the economy produces multiple goods, and these goods are *complements*, then explain - in words - what happens to the share of expenditure on a good when firms get more productive at producing that good.

   [**Answer**]: If the goods are complements, then when productivity of good X goes up, it becomes cheaper to buy X. But, because people don't want to substitute towards X, they take advantage of the cheaper X to spend less (lower their expenditure share) on X, and instead use the extra money to buy more of other goods.

7. **[5 points]** If the true production function is $Y = K^{\alpha}(EL)^{1-\alpha}$, then under what conditions will a *residual* defined by $Res = Y/K^{1-s_L}L^{s_L}$, where s_L is the share of output earned by labor, be equal to E? By conditions, I mean more than just a statement about parameters, I mean a statement about how markets work.

   [**Answer**]: Only if factor markets are efficient and firms are competitive, meaning that the markup is equal to one (price = marginal cost), will Res = E.

8. **[5 points]** In a model with two capital types, and production in per-worker terms of $y_t = k_t^{\alpha} z_t^{\beta}$, where $\alpha + \beta < 1$, and constant savings rates for both capital types (s_k and s_z), can the steady state growth rate be different than zero? Explain.

   [**Answer**]: No. The values $\alpha + \beta < 1$. So no matter how much we save, as we accumulate more of both capital types, output does not keep up. That is, if we double our stock of capital, we do not double our output per capita. On the other hand, the depreciation of that capital has doubled, so we cannot keep up. Eventually, growth in output per capita has to go to zero.

9. **[5 points]** An individual lives two periods. They have wages w_1 in the first period of life, and zero wages in the second period. Their intertemporal elasticity of substitution is less than one (i.e. $\sigma > 1$). What is the effect of an increase in r on their consumption in period 1, c_1?

   [**Answer**]: With no wealth effect, all we have to worry about is income and substitution. With EIS less than one, we know the income effect dominates, meaning that the increase in r means I can raise consumption in period 1. In math, with $\sigma > 1$, then the solution of the two period problem shows that $\partial c_1/\partial r > 1$.