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A Theory of Structural Change That Can Fit the Data†

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We study structural change in the historical consumption expenditure of the United States, the United Kingdom, Canada, and Australia over more than a century. We characterize the most general class of preferences in a time-additive setting that admits aggregation of the saving decision and allows us to identify preference parameters from aggregate data. We parameterize and estimate such intertemporally aggregable (IA) preferences and discuss their properties in a dynamic general equilibrium framework with sustained growth. Our preference class is considerably more flexible than the Gorman form or PIGL, giving rise to a good fit of the non-monotonic pattern of structural change. (*JEL* C51, E21, L16, N10)

As countries develop, the consumption expenditure and value-added shares of the agricultural sector tend to decline steadily, the share of manufacturing first increases and then decreases, and eventually services become the dominant sector. Qualitatively, this is a robust pattern across time and space. In this paper, we make three contributions to the structural change literature. First, we document this robust pattern of structural transformation in the United States (USA), the United Kingdom (GBR), Canada (CAN), and Australia (AUS) with new consumption expenditure data covering over a century. Second, we analyze structural change in a multisector growth model and characterize the most general class of preferences for which aggregate expenditure and saving are independent of inequality—a property that we call *intertemporal aggregation*. Third, we show that this demand structure allows us to consistently estimate the preference parameters from aggregate sectoral expenditure data and that its flexibility is required to fit the data.

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Although the pattern of structural change is well documented in other data, the empirical literature has come to different conclusions on whether stable preferences are consistent with this pattern. Herrendorf, Rogerson, and Valentinyi (2013) find that the standard generalized Stone-Geary preferences can match the USA's structural change in the postwar era—using both the final consumption expenditure and the consumption component of value added. In contrast, Buera and Kaboski (2009) show, for historical value-added data starting in 1870, that the same preferences struggle to fit the data for the USA. However, as constructing the consumption component of value added requires input-output tables that are not available for the prewar period, the results in Buera and Kaboski (2009) and Herrendorf, Rogerson, and Valentinyi (2013) are not directly comparable.

In this paper, we focus on structural change from the perspective of final consumption expenditure, where sectoral consumption data for both the prewar and postwar periods are directly available for the USA, GBR, CAN, and AUS. Three strong and robust regularities emerge from the data across all four countries: a continued decline of the expenditure share for agriculture, a hump-shaped manufacturing share, and an accelerating rise of the service share, both over time and in real per capita income. Most studies of structural change that quantify demand forces have restricted the analysis to the postwar period, which would not reveal the regularities in the hump-shaped manufacturing share and the accelerating rise of the service share in our sample, as manufacturing is steadily declining and services steadily increasing since the 1950s.

The non-monotonic pattern of the expenditure shares described above calls for non-homothetic preferences with flexible income effects, such that the marginal propensity to consume a particular good changes with income (i.e., preferences that are outside the Gorman form). This limits the tractability in dynamic general equilibrium models because inequality affects the aggregate demand structure, and there is no strict representative consumer. As a result, it is challenging to make welfare statements and to identify preference parameters from aggregate data.[1]

We propose a new class of preferences that combines flexible income effects with tractable aggregation. In our theoretical framework with time-additive preferences, the household problem can be split into two decisions: the optimal savings decision (the intertemporal problem) and how to spend total expenditure in a period on different sectors (the intratemporal problem). Our proposed class restricts preferences such that aggregate saving and expenditure are independent of inequality; we call this property intertemporal aggregation and characterize the full class of such preferences. The preferences in our class imply that the marginal utility of any household relative to the household with the average expenditure level remains constant over time. The impact of inequality on the aggregate sectoral consumption demand is then reduced to a simple scalar. As a consequence, all parameters can be estimated from aggregate data, up to one constant that can be identified from information on the expenditure distribution at one point in time. Despite this intertemporal aggregation property, the functional form allows for differences across households in the

[1] A quantitatively valid framework is crucial to assess the welfare effects of structural change. For example, income effects can reinforce or dampen the productivity slowdown from the Baumol (1967) cost disease.

marginal propensity to consume from specific sectors within a period, i.e., inequality matters for the intratemporal expenditure structure.

The resulting class of intertemporally aggregable (IA) preferences is parsimonious and flexible. For example, at given prices, a specific good can be a luxury for low income levels and a necessity for high levels. In cross-sectional microeconomic data, we document precisely this pattern for manufacturing.[2] We show that our IA class directly nests the frequently used generalized Stone-Geary and the Price-Independent Generalized Linearity (PIGL) preferences (see Muellbauer 1975, 1976) as special cases. The additional flexibility is required to fit the non-monotonic pattern of structural change. We demonstrate that the IA specification—despite its flexibility—is consistent with a standard multisector growth model as put forward by Herrendorf, Rogerson, and Valentinyi (2014), i.e., it supports an asymptotic balanced growth path with an arbitrary number of sectors.

In the quantitative analysis, we estimate a simple parameterization of our IA preferences for the historical sample, which includes the prewar period, and compare its fit with the one of the nested generalized Stone-Geary and PIGL specifications. We find that IA preferences can fit the data and are able to generate the non-monotonic pattern of structural change. In particular, IA preferences have the necessary flexibility to fit the hump-shaped manufacturing share because they allow manufacturing to be a luxury at the beginning of the sample and a necessity toward the end. Furthermore, IA preferences allow for sustained income effects, which enables agriculture to be a strong necessity throughout the sample period. By contrast, the income effects of the generalized Stone-Geary specification converge monotonically to zero as income increases. It therefore struggles to fit the strong empirical regularities outlined above.[3] Like the IA class, PIGL preferences permit sustained income effects, and this allows them to fit the continued decline in agriculture and the acceleration in services at high per-capita income levels. However, PIGL preferences do not allow income effects to be flexible, and consequently, they cannot fit the non-monotonic pattern as well. Overall, we find that IA preferences provide the best fit for the individual countries and the pooled sample.

The remainder of the paper is organized as follows. Section I describes the historical panel data and establishes the empirical regularities. In Section II we present the general theoretical framework, and in Section III we characterize the class of IA preferences. Section IV presents a simple parameterization of preferences and Section V contains the structural estimation and discusses the main empirical results. Section VI relates our study to the existing literature and provides practical guidance for applied users of our preferences. Section VII concludes. All proofs and additional lemmata and estimation tables are in Appendix A. Additional material and a detailed description of the historical data are delegated to the online Appendix.

[2] See also Banks, Blundell, and Lewbel (1997), who show that this is generally an essential feature of microeconomic data.

[3] This finding is in line with the conclusion in Buera and Kaboski (2009), which is, however, based on value-added data while we focus on final consumption expenditure. Buera and Kaboski (2009) assume that for agriculture and services, sectoral consumption corresponds to sectoral value added because historical input-output tables are not available. Manufacturing consumption is constructed by deducting all final investment from manufacturing value added.

I. Historical Data on Structural Change

The distinguishing feature of our novel dataset is that it provides consistent sectoral prices and consumption expenditure for four countries over more than a century. The selection of the four countries—USA, GBR, CAN, and AUS—is determined by the availability of historical data with sufficiently detailed expenditure and price categories including the prewar period.

A. *Data Sources and Coverage*

We obtain the data from the national statistical offices whenever available and complement them with historical data from Carter et al. (2006) for the USA, Feinstein (1972) for GBR, and Haig and Anderssen (2006) for AUS. For CAN the single data source is Statistics Canada.[4] The data for USA, GBR, and AUS cover the period 1900–2014, and the data for CAN cover the period 1926–2014. We exclude years when a country was involved in World Wars I and II or severely affected by the Great Depression because of our focus on long-run trends. This also addresses concerns regarding the data quality during these years.

We use the detailed nominal final expenditure and price data for all four countries and aggregate the fine consumption categories to three broad sectors: agriculture, manufacturing, and services.[5] Roughly speaking, agriculture consists of food and beverages purchased for off-premise consumption. Manufacturing includes durable goods, clothing and footwear, gasoline and other energy goods, and other nondurable goods. Services consists of private services consumption, but, in a robustness check, we also include government consumption. This categorization follows Herrendorf, Rogerson, and Valentinyi (2013) and is standard in the structural change literature. The resulting sectoral price indexes are adjusted for the local currency and purchasing power parity (PPP) to ensure that the real quantities are in the same units across countries.[6]

B. *Final Consumption Expenditure Shares*

Figure 1 illustrates three robust regularities of structural change in the USA, GBR, CAN, and AUS since the beginning of the last century. First, panel A shows that there has been a steady decline in the expenditure share of agriculture. Historically, agriculture used to be the largest sector. For example, in the USA, the share of food and beverages in private consumption fell from 41 percent to only 7 percent during our sample period, as can be seen from panel D. Second, panel B illustrates that the expenditure share of manufacturing consumption is hump-shaped over time. Again, using the USA as an example, the share of manufacturing was 24 percent in

[4] The data from Carter et al. (2006) are based on Lebergott (1996). All the data sources and the categorization of the sectors are described in online Appendix C.

[5] We use Fisher indexes to aggregate prices and quantities of the detailed consumption categories to the three broad sectors. The details are explained in Section C2 of the online Appendix.

[6] We use the PPP conversion factors for the year 1990 provided by the World Bank (2016) in the World Development Indicators (WDI). See Section C3 of the online Appendix for further details.

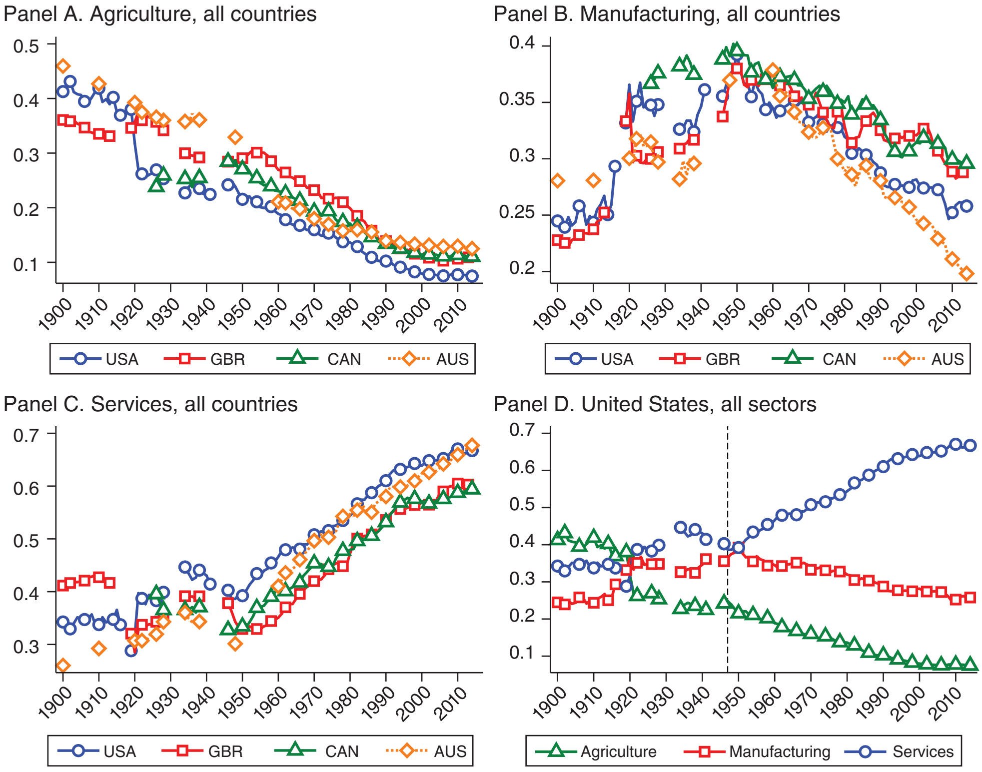

Figure 1. Final Private Consumption Expenditure Shares

Notes: The figure plots the final private consumption expenditure shares over time for all countries. Panels A–C plot the shares by sector, and panel D shows all shares for the USA separately. The years affected by World War I, World War II, and the Great Depression are excluded.

Source: See online Appendix C.

1900, then reached its peak of 39 percent in 1950, and, finally, declined gradually to 26 percent by the end of the sample. Third, panel C shows an accelerating rise of the service sector. The share of services increased moderately between 1900 and 1950 (from 34 percent to 39 percent in the USA) and then more rapidly (to 67 percent) in the second half of the sample.

Similar regularities have been documented for other countries and complementary measures of structural change (see, for example, Buera and Kaboski 2012; Uy, Yi, and Zhang 2013; Herrendorf, Rogerson, and Valentinyi 2014; and Comin, Lashkari, and Mestieri 2021 for recent contributions). Furthermore, we see the same regularities in expenditure shares when we plot them against real per capita GDP.[7]

[7] This is illustrated in Figure B1 of the online Appendix, where we plot the expenditure shares against the real per capita GDP taken from Bolt and van Zanden (2014). To test the pattern more formally, we also regressed the sector shares on log real per capita GDP. Following Buera and Kaboski (2012), we split the sample at the real per capita GDP level that corresponds to the peak in manufacturing. The coefficients in each subsample confirm the above regularities.

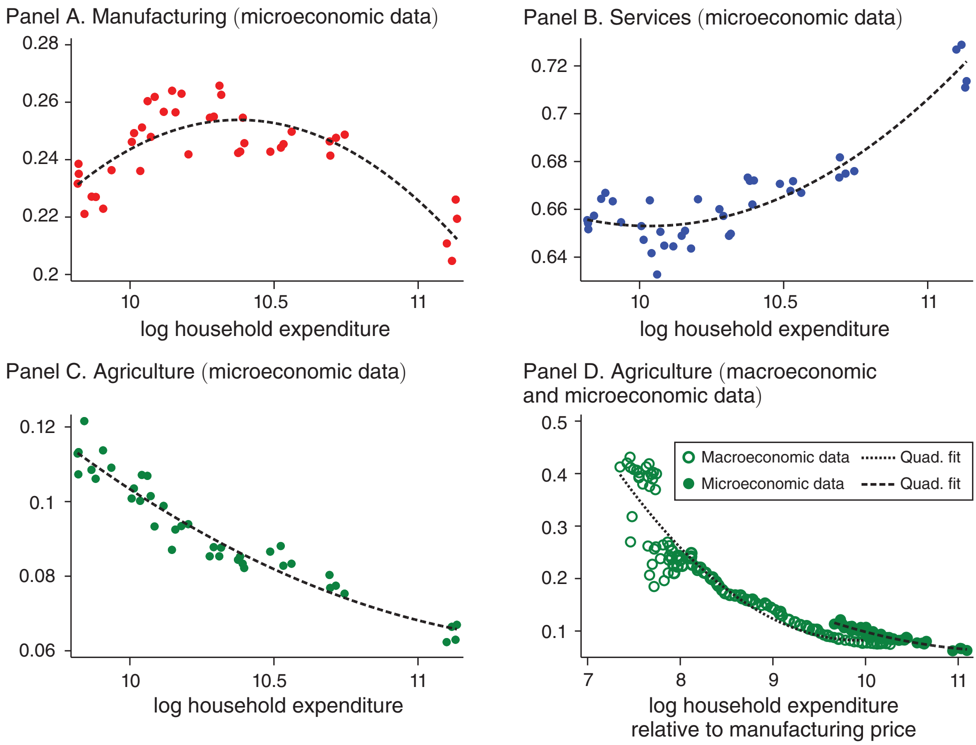

Figure 2. Consumption Expenditure Shares across US Households

Notes: The figure plots the consumption expenditure shares of agriculture, manufacturing, and services against total household expenditure for the years 2014–2017 in the USA. In each year, households are grouped by income deciles and each dot in the figure represents the average household expenditure of the income group in that year. The dashed line is a quadratic fit. We adjust expenditure for differences in household size using the OECD-modified equivalence scale. Differences in the average expenditure levels across the four years are removed by controlling for year fixed effects. Panel D combines the microeconomic data with the macroeconomic time-series data.

Source: See online Appendix C.

The pattern of structural change in the aggregate is qualitatively consistent with recent microeconomic expenditure data from the US Bureau of Labor Statistics (2022). Figure 2 shows the expenditure shares of the same consumption categories when plotted against the level of total household expenditure from 2014 to 2017 (adjusting for household size and controlling for year fixed effects). Panels A–C show that the patterns in the macroeconomic and microeconomic data are strikingly similar. As illustrated in panel D, for agriculture, the gradients of the expenditure shares are even quantitatively comparable.[8] As the cross-sectional data isolate the income effects (at constant prices), this pattern suggests that non-homothetic preferences are necessary to fit the data. Furthermore, the income effects need to be

[8]The remaining sectors are shown in Figure B2 of the online Appendix. In panel D of Figure 2 and in all panels of Figure B2, we scaled the total household expenditure to match the level in 2014, when we observe both macro- and microeconomic data. Furthermore, we express total household expenditure in terms of the manufacturing price to account for price changes over time in the macroeconomic data.

flexible to fit the hump-shaped manufacturing share, which is an essential feature of the preference class we introduce further below. However, there are also some quantitative differences between the microeconomic and macroeconomic data; for example, the manufacturing share peaks at a different level. This is consistent with relative prices—besides income effects—playing a significant role for the observed structural change in the aggregate.

C. *Relative Prices and Per Capita Expenditure*

This section documents the evolution of relative prices, quantities, and per capita expenditure in the historical data. In principle, the structural change over the last century could be completely driven by changes in relative prices. However, our data show that price effects need to be complemented with sustained and flexible income effects to account for the patterns in Figures 1 and 2.

Why are income effects needed? Figure 3, panels A and B plot the prices of agriculture and services relative to manufacturing on a ratio scale. All relative prices are normalized to unity in the year 1927. The sectoral prices relative to manufacturing remained relatively stable in the first half of the sample and then started to increase around 1950. The price increase is more pronounced for services than for agriculture, and—if services are a sufficiently strong complement—the relative price alone could explain the late rise of the service sector documented earlier. However, for the agricultural sector, both the price and real consumption relative to services are falling over time since 1950. With homothetic preferences, not even perfect complements can explain such a positive relationship. Hence, in addition to relative price effects, income effects are needed to explain the historical structural change.[9]

Why are *flexible* income effects needed? Figure 3, panel C shows that the price and quantity of agriculture relative to services fall together for more than 60 years in the USA, while in the first half of the sample, relative prices and quantities of agriculture are negatively related overall. Since per capita expenditure is steadily growing at the same time, this suggests that agriculture must have a substantially lower income elasticity of demand relative to the service sector in the postwar period compared to the prewar period. Hence, it is not sufficient to have income effects; they must also be flexible. Such flexibility is also required to be consistent with the microeconomic data presented in Figure 2. The hump shape of the manufacturing expenditure share in Figure 2, panel A implies that, for given sectoral prices, manufacturing is a luxury for poorer households while it is a necessity for the rich. Such a pattern is impossible to generate with generalized Stone-Geary or PIGL preferences, for example.

Finally, Figure 3, panel D illustrates that there has been sustained per capita expenditure growth in all four countries (with the exceptions of GBR and AUS between 1900 and 1920).[10] Note that per capita expenditure is plotted on a ratio

[9] A similar argument can be made with manufacturing and services, for which both the relative price and quantity have been falling (see Figures 2 and 3 in Boppart 2014).

[10] Note that real per capita expenditure is unobserved in the data. Thus, in the figure, we proxy real expenditure by expressing nominal expenditure relative to the price of manufacturing. The qualitative conclusions from the

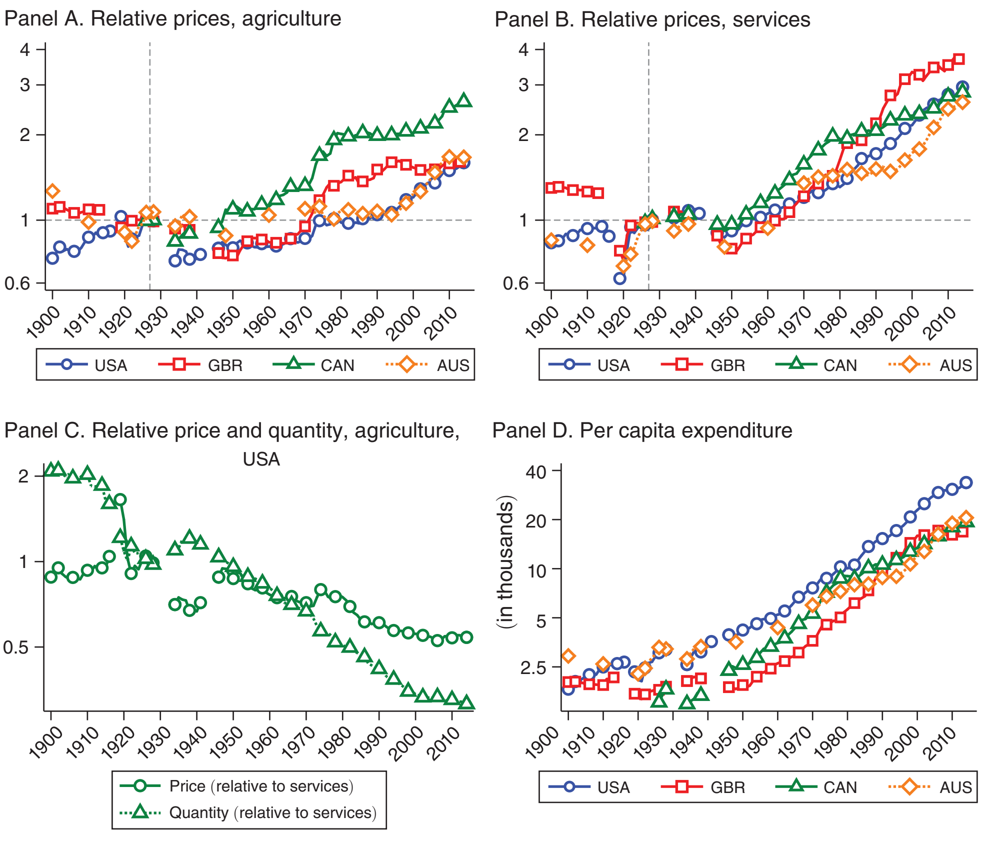

Figure 3. Relative Prices and Private Per Capita Consumption Expenditure

Notes: Panels A and B plot the prices of agriculture and services relative to manufacturing over time for all countries, and panel D plots the nominal per capita expenditure relative to the manufacturing price. All nominal variables are based on final private consumption expenditure and expressed in PPP-adjusted 1990 international dollars. In panels A and B, relative prices are normalized to unity in 1927 and plotted on a ratio scale. Panel C shows the price and quantity of agriculture relative to services in the USA. In panel D, per capita expenditure is plotted on a ratio scale. The years affected by World War I, World War II, and the Great Depression are excluded.

Source: See online Appendix C.

scale; thus, the slope approximates the yearly growth rate. For the USA, for example, relative per capita expenditure has increased by more than a factor of 18 between 1900 and 2014. With income effects, the enormous increase in per capita expenditure can potentially play an important role in explaining the pattern of structural change over the last century.

figure remained unchanged if we used, for example, a Fisher index over the sectoral prices to deflate the nominal expenditure.

II. Theoretical Framework

In this section, we present the theoretical framework in which we analyze structural change. The production side of our framework coincides with the "benchmark model" in Herrendorf, Rogerson, and Valentinyi (2014). On the consumer side, however, we keep preferences general and allow for heterogeneity in consumers' factor endowments. In Section III, we then discuss the properties of specific preference specifications in our framework.

A. *Economic Environment*

We consider an infinite-horizon, closed-economy framework in discrete time with four production sectors. Our main focus is on the three consumption sectors called agriculture A, manufacturing M, and services S, but we also explicitly model a fourth sector that produces an investment good X. In each sector $j \in J_+ \equiv \{A, M, S, X\}$, output $y_{j,t}$ is competitively produced according to the following Cobb-Douglas technology:

$$y_{j,t} = k_{j,t}^{\alpha} \left(g_j^t n_{j,t}\right)^{1-\alpha}. \tag{1}$$

Here, $k_{j,t}$ and $n_{j,t}$ denote capital and labor used in sector j, and g_j^t is a Harrod-neutral technology term (where t denotes time). The initial technology term is normalized to one in all sectors. We assume $\alpha \in (0,1)$ and $g_j \geq 1, \ \forall j$.[11] Firms in all sectors take the rental rate, $R_t = r_t + \delta$, the wage rate, w_t, and the output price, $p_{j,t}$, as given and then choose their capital and labor input to maximize profits. The capital and labor market clearing requires

$$\sum_{j \in J_+} k_{j,t} = k_t, \quad \text{and} \quad \sum_{j \in J_+} n_{j,t} = n, \tag{2}$$

where k_t and n denote total capital and labor in the economy.

The output of agriculture, manufacturing, and services is consumed, whereas the output of sector X is invested. There is an interval of infinitely lived households indexed by $i \in [0, N]$ with the following preferences (where $\beta \in (0,1)$ denotes the discount factor):

$$\mathcal{U}_{i,0} = \sum_{t=0}^{\infty} \beta^t v\left(e_{i,t}, P_t\right), \qquad P_t \equiv \left(p_{A,t}, p_{M,t}, p_{S,t}\right). \tag{3}$$

The period utility function $v(e_{i,t}, P_t)$ is given in *indirect form*, i.e., it is defined over nominal expenditure $e_{i,t}$ and the vector P_t of prices of all consumption goods.[12] For our intertemporal application, we assume that $v(\cdot)$ is three times continuously differentiable in e and continuously differentiable in all prices, and that we have

[11] Furthermore, we assume that $g_X > 1$, such that capital can be accumulated at a sustained positive rate. All sectors produce with a Cobb-Douglas technology over capital and labor, and there is no technological regress. This implies that the output of all sectors can grow at a steady positive rate, as well.

[12] We assume that this function $v(\cdot)$ fulfills standard regularity conditions, i.e., is strictly decreasing in all prices, strictly increasing in e, quasi-convex, and homogenous of degree zero in all prices and e.

$v_{ee}(e_{i,t}, P_t) < 0$. We allow for heterogeneity across households in their inelastically supplied labor units $n_i \geq 0$ and in their level of initial wealth $a_{i,0}$. As preferences are additively separable over time, the household's problem can be split up into an intertemporal and an intratemporal problem. The intertemporal problem deals with the optimal saving/spending decision, i.e., choosing a sequence $\{e_{i,t}, a_{i,t+1}\}_{t=0}^{\infty}$ to maximize (3) subject to

$$a_{i,t+1} = a_{i,t}(1 + r_t) + w_t n_i - e_{i,t}, \tag{4}$$

and a standard no-Ponzi game condition.[13] For the intratemporal problem, applying Roy's identity to the indirect utility function gives the Marshallian demands $c_{i,j,t}$, $j \in J \equiv \{A, M, S\}$ that describe how nominal expenditure, $e_{i,t}$, is spent on the three consumption sectors.

We choose the investment good as the numéraire, $p_{X,t} = 1$, $\forall t$. The choice of numéraire implies that e, w, and r in this Section II should be understood as expressed in units of investment goods. In the following, we refer to this e as simply expenditure. The asset and labor market clearing conditions read

$$\int_0^N a_{i,t}\, di = k_t, \quad \text{and} \quad \int_0^N n_i\, di = n, \tag{5}$$

and the law of motion of aggregate capital becomes $k_{t+1} = k_t(1 - \delta) + y_{X,t}$, where $\delta \in [0,1]$ is the depreciation rate. Clearing of the consumption sectors requires

$$\int_0^N c_{i,j,t}\, di = y_{j,t}, \quad \forall j \in J. \tag{6}$$

In macroeconomic theory, it is more common to work with direct utility functions instead of the indirect formulation used here. However, as we will see below, the indirect formulation allows us to characterize the optimal saving decision as simply as in a one-sector economy. This enables us to highlight the additional restrictions that the existence of a balanced growth path imposes on preferences. Furthermore, in Section III, we characterize the most general class of preferences in a time-additive setting that admits aggregation of the saving decision, and this general class of preferences only admits a closed form for the indirect utility function (whereas the direct formulation may only be implicitly defined). We therefore prefer to work here with the indirect formulation. Note, however, that the empirically observed object is the implied demand system, which is identical for both the direct and indirect formulation. In general, the direct utility function $u(\cdot)$ can be defined implicitly by the following system:

$$u(c) = v(e, z(c)), \tag{7}$$

$$c_j = -\frac{\partial v(e, z(c))/\partial z_j(c)}{v_e(e, z(c))}, \quad \forall j \in J, \tag{8}$$

[13] The no-Ponzi game condition can be expressed as $\lim_{T\to\infty} a_{i,T+1} \prod_{s=1}^{T}(1 + r_s)^{-1} \geq 0$.

where $c = (c_A, c_M, c_S)$ and $z(c) = (z_A(c), z_M(c), z_S(c))$ are vectors and e can be normalized to one. For the economy above, where relative prices are entirely determined by technology, we show in Section AA of the Appendix a compact way to state the planner problem. Moreover, for the parameterized class of preferences that we estimate using our historical data, we will restrict parameters such that a closed-form direct utility function exists and specify its functional form in Proposition 4.

Although we are interested in structural change between different consumption good sectors, we nevertheless model the investment good as a separate sector as opposed to, e.g., assuming that all investment comes from the manufacturing sector.[14]

B. *Equilibrium Definition and Discussion*

We will, in the following, focus on the competitive outcome of our dynamic general equilibrium framework and compare its prediction to the historical consumption expenditure data of Section IB. We define an equilibrium as a sequence of prices and quantities that is jointly consistent with utility maximization of all households, profit maximization (and perfect competition) of all firms, as well as the market clearing conditions (5) and (6).

Although the dynamic framework is, in some sense, very standard, it seems relevant to comment here on its generality. First, our focus on a decentralized market equilibrium is not central, as the competitive equilibrium is Pareto efficient (and could also be characterized as the solution to a planner's problem). Second, the framework is flexible enough to allow for changing relative prices between sectors. It also explicitly models capital accumulation, and consistency with a path of sustained and balanced growth can be discussed. Third, note that the imposed restrictions on the preference side, like time additivity and discounting, are relatively mild and standard, and we keep at this point full flexibility with respect to the period utility. On the production side, however, the framework puts some simplifying structure; most importantly, it assumes identical output elasticities of capital α across the three consumption sectors (as well as the investment good sector).[15] This precludes factor intensity differences as a source of relative price changes (*à la* Acemoglu and Guerrieri 2008), and that shifts in the demand structure due to income effects have an impact on relative prices (see Caselli and Coleman 2001). The Cobb-Douglas form of production could be relaxed, and the capital intensity

[14] Hence, our theory can accommodate investment-specific technical change. See García-Santana, Pijoan-Mas, and Villacorta (2021) and Herrendorf, Rogerson, and Valentinyi (2021) for theories of structural change between and in investment and consumption.

[15] Without identical capital intensities across the consumption sectors, the technology side of the economy would already exclude the coexistence of structural change with an exact, balanced growth path. The assumption of equal factor intensities seems empirically justifiable at least for the capital-labor split across different consumption sectors. Valentinyi and Herrendorf (2008) report for the USA in the year 1997 similar labor shares of 0.34 and 0.35 for the services and for total consumption, respectively. Finally, Herrendorf, Herrington, and Valentinyi (2015) argue, based on a production function estimation, that Cobb-Douglas technologies with identical output elasticities of capital but different TFP growth capture the main technological forces behind structural change for the postwar USA.

could be allowed to differ between the consumption sectors and the investment sector. These generalizations would not affect the model's main predictions.

C. *Equilibrium Implications*

As production differs only in the labor-augmenting technology terms across sectors, prices are solely pinned down by technology, and we have $p_{j,t} = \left(g_X/g_j\right)^{(1-\alpha)t}, \ \forall j \in J$. Output in each sector can then be written as a linear function of its labor used, i.e., $y_{j,t} = g_j^{(1-\alpha)t}\left(k_t/n\right)^{\alpha} n_{j,t}, \ \forall j \in J_+$. All equilibrium conditions are formally derived in Section AB of the Appendix.

Time-varying rates of technical change—in particular, in the investment sector—would ex ante rule out the existence of a balanced growth path. Imposing that the rates of technical change eventually converge to a (sector-specific) constant is a relatively mild restriction. In order to discuss preferences' consistency with exact balanced growth, however, we assume constant rates of technical change not only asymptotically but throughout. This allows relative prices to change over time but restricts these changes to happen at constant rates. This is a good first-pass approximation of the postwar data but not of the full sample period (see Section IC). Hence, the concept of exact balanced growth should be understood as mainly bearing potential relevance post-World War II. When we estimate preference parameters in Section V, we take the prices in the data as given, and the assumption of constant rates of technical change is inconsequential.

The optimal saving behavior of a household i is characterized in the following lemma.

LEMMA 1: *Solving the intertemporal household problem gives rise to the Euler equation*

$$\frac{v_e\left(e_{i,t}, P_t\right)}{v_e\left(e_{i,t+1}, P_{t+1}\right)} = \beta\left(1 + r_{t+1}\right), \tag{9}$$

where $v_e\left(e_{i,t}, P_t\right)$ is the indirect marginal utility of expenditure in a given period.

PROOF:

In Section AC of the Appendix. ∎

Jointly with the budget constraint, (4), the transversality condition, and the initial wealth $a_{i,0}$, this Euler equation fully characterizes the household's saving behavior. Aggregating all the household budget constraints and combining them with (5) gives

$$k_{t+1} = k_t\left(1 - \delta\right) + k_t^{\alpha}\left(g_X^t n\right)^{1-\alpha} - E_t, \tag{10}$$

where $E_t \equiv \int_0^N e_{i,t}\,di$ is aggregate expenditure. This allows us to characterize the dynamics of the capital stock and finally solve the model.

In the following, we are interested in the long-run properties of the equilibrium path. To this aim, we next define the concept of balanced growth.

DEFINITION 1: *A balanced growth path is an equilibrium path along which the aggregate physical capital stock k_t grows at a constant positive rate. If such a balanced growth path can be reached with a finite capital stock, then we call it an exact balanced growth path. If the balanced growth path only exists as the capital stock approaches infinity, then we call it an asymptotic balanced growth path.*

Similar to Kongsamut, Rebelo, and Xie (2001) and Herrendorf, Rogerson, and Valentinyi (2014), we use a generalized notion of balanced growth where sectoral variables are not restricted to grow at constant rates. The production side is potentially in line with an (exact) balanced growth path. A balanced growth path exists if the Euler equation (9) is jointly consistent with a constant interest rate, r_{t+1}, and a constant expenditure growth rate in terms of investment goods, $e_{i,t+1}/e_{i,t}$, either asymptotically or for a finite expenditure level. Hence, whether the economy admits a balanced growth path depends on the specified period utility function.[16] As long as preferences are well specified, *asymptotic balanced growth* is generally fulfilled as each expenditure share converges to a constant.

Intratemporal optimality, i.e., how to spend a given expenditure level on the different sectors, is obtained by applying Roy's identity to $v(e_{i,t}, P_t)$, yielding the Marshallian demands. The functional form of this demand system depends on the precise formulation of the period utility function. In the next section, we ask what restriction must be imposed on the function $v(\cdot)$ such that preferences preserve that the intertemporal problem can be aggregated. We then characterize the full class of such preferences and show that it accommodates frequently used formulations as special cases.

III. A General Class of Preferences

Flexible demand systems typically do not admit Gorman aggregation and, in general, the preference parameters cannot be estimated from aggregate data without bias. How can we consistently retrieve preference parameters without restricting the utility class too much? Our approach is to rely on the dynamic framework in Section II, restrict preferences such that aggregation in the intertemporal dimension is preserved, and then show how this allows us to identify preference parameters from aggregate data.

A. *Intertemporal Aggregation*

We now define the class of intertemporally aggregable (IA) preferences.

DEFINITION 2: *Consider our framework with time-additive preferences of the form $\mathcal{U}_{i,0} = \sum_{t=0}^{\infty}\beta^t v(e_{i,t}, P_t)$ and intertemporal optimization such that the Euler equation (9) holds for each household. We call preferences $\mathcal{U}_{i,0}$ intertemporally*

[16] In Section AD of the Appendix we formally show that if a balanced growth path exists, then its dynamics are fully determined by the exogenous rates of technical change.

aggregable (*IA*) *if average per capita expenditure* E_t/N *satisfies the individual Euler equation irrespective of the cross-sectional expenditure distribution*, *i.e.*, *we have*

$$\frac{v_e(E_t/N, P_t)}{v_e(E_{t+1}/N, P_{t+1})} = \beta(1 + r_{t+1}), \quad \forall P_t, P_{t+1}, r_{t+1}. \tag{11}$$

The Euler equation (9) describes the law of motion of all individual expenditure levels $e_{i,t}$ as a function of the interest rate, the rate of time preference, and prices. Aggregating all expenditure levels up then gives the path of the aggregate (and per capita) expenditure level. As stated in Definition 2, preferences are IA if this path of average per capita expenditure itself again satisfies the Euler equation—independent of the distribution of individual expenditure. This aggregation property implies that the economy admits *intertemporally* a representative agent.

Although IA preferences admit a representative agent for the intertemporal consumption/saving decision, they still allow for considerable flexibility of the intratemporal income effects, which is essential to match the data. Note also that the definition of IA does not restrict expenditure levels $e_{i,t}$ to grow at identical rates; the Euler equation restricts the *marginal utility*, $v_e(\cdot)$, to grow at the same rate across households at a given point in time. This can be consistent with convergence or divergence in the distribution of expenditure levels.[17]

In the next proposition, we fully characterize the class of period utility functions that allow for intertemporal aggregation according to Definition 2.

PROPOSITION 1: *Preferences* (3) *are intertemporally aggregable if and only if the period utility* $v(e_i, P)$ *takes* (*up to multiplicative or additive constants*) *one of the following forms:*

$$v(e_i, P) = \frac{1-\epsilon}{\epsilon}\left(\frac{e_i}{\mathbf{B}(P)} - \mathbf{A}(P)\right)^{\epsilon} - \mathbf{D}(P), \qquad \epsilon \notin \{0,1\}, \tag{12}$$

$$v(e_i, P) = -\exp\left(-\left(\frac{e_i}{\mathbf{B}(P)} - \mathbf{A}(P)\right)\right) - \mathbf{D}(P), \tag{13}$$

or

$$v(e_i, P) = \mathbf{F}(P)\log\left(\frac{e_i}{\mathbf{B}(P)} - \mathbf{A}(P)\right), \tag{14}$$

where $\mathbf{A}(P)$, $\mathbf{D}(P)$, *and* $\mathbf{F}(P)$ *are functions homogenous of degree zero in prices, and* $\mathbf{B}(P)$ *is a linearly homogenous function of prices.*

PROOF:

In Section AE of the Appendix. ∎

[17] IA is, therefore, a weaker restriction than the mean scaling discussed in Lewbel (1989).

The proof of Proposition 1 starts by showing that IA requires $e_{i,t+1}$ to be an affine-linear function of $e_{i,t}$ with coefficients that may depend on prices. The intertemporal Euler equation can then be differentiated twice, rearranged, and integrated up twice to get the above restrictions on the utility function.

Given the general restriction in Definition 2, the resulting period utility function is parsimonious, fairly flexible with three non-redundant price functions, and nests (as we will show below) some well-known cases. In the special case of one commodity, we obtain the class of the "hyperbolic absolute risk aversion" (HARA) period utility function. This one-commodity HARA case is well known to be the most general form of the period utility such that overall preferences $\mathcal{U}_0$ are part of the Gorman class in a time-additive setting.[18] However, the class of Gorman preferences is clearly too restrictive to fit the historical data. Proposition 1 broadens this class but still preserves a useful aggregation result in our intertemporal framework.

Proposition 1 states the necessary and sufficient conditions for intertemporal aggregation. Further restrictions need to be imposed on the price functions to satisfy the regularity conditions of the period utility function and to ensure an interior solution of the intertemporal problem. We discuss these issues when we parameterize the preferences further below. Note that Definition 2 implicitly assumes that the Euler equation characterizes the individual choice. Hence, similar to existing models of structural change, we abstract from frictions in the saving decision.

The next proposition establishes the Marshallian demand system of IA preferences.[19]

PROPOSITION 2: *If preferences are IA with period utility function* (12) *or* (13), *then the Marshallian demand of each commodity j is given by*

$$c_{i,j,t} = \mathbf{A}_j(P_t)\mathbf{B}(P_t) + \frac{\mathbf{B}_j(P_t)}{\mathbf{B}(P_t)} \cdot e_{i,t} + \frac{\mathbf{D}_j(P_t)}{v_e(e_{i,t}, P_t)}, \tag{15}$$

where $\mathbf{A}_j(P_t)$, $\mathbf{B}_j(P_t)$, *and* $\mathbf{D}_j(P_t)$ *denote derivatives of the corresponding functions with respect to* $p_{j,t}$. *In per capita terms,* $C_{j,t}/N \equiv 1/N\int_0^N c_{i,j,t}di$, *the Marshallian demand of each commodity is given by*

$$C_{j,t}/N = \mathbf{A}_j(P_t)\mathbf{B}(P_t) + \frac{\mathbf{B}_j(P_t)}{\mathbf{B}(P_t)} \cdot E_t/N + \kappa\frac{\mathbf{D}_j(P_t)}{v_e(E_t/N, P_t)}, \tag{16}$$

where the time-constant aggregation factor κ *is given by*

$$\kappa \equiv \frac{1}{N}\int_0^N \frac{v_e(E_t/N, P_t)}{v_e(e_{i,t}, P_t)}di. \tag{17}$$

[18] See Pollak (1971) for a proof of this result. It is easy to show that even for our multiple commodity case the coefficient of absolute risk aversion becomes a hyperbolic function in e_i.

[19] In Proposition 2, we focus on the IA preferences with period utility function (12) or (13). This is the demand system that we consider in the empirical application below. For completeness, we also state the Marshallian demand system of function (14) in equations (A26) and (A27) of Appendix A. All theoretical results established in this section generalize to IA preferences with the period utility function (14).

PROOF:

In Section AF of the Appendix. ∎

The IA demand system in equation (15) contains three distinct additive functions of expenditure $e_{i,t}$.[20] This implies flexible income effects, i.e., a non-monotonic relationship between $e_{i,t,}$ and the expenditure shares. For instance, the demand system can generate hump-shaped expenditure shares in $e_{i,t}$. Banks, Blundell, and Lewbel (1997) establish that matching microeconomic data typically requires this flexibility. Our class nests several standard preferences often used in the literature, but these lack the flexibility to generate non-monotonic expenditure shares.[21]

Proposition 2 also establishes that up to a constant κ, which scales the last term in (16), the individual demand and the aggregate per capita demand take an identical structure. In the presence of heterogeneity in individual expenditure, κ differs from one. Working under a representative agent assumption would then lead to an aggregation bias as the individual demand evaluated at $e_{i,t} = E_t/N$ differs from (16).[22] We formalize this property in the following corollary that generalizes Theorem 7 in Muellbauer (1975) to IA preferences.

COROLLARY 1: *If the distribution of $v_e(E_t/N, P_t)/v_e(e_{i,t}, P_t)$ is constant over time, then IA is the most general preference specification for which, given knowledge of the distribution of $e_{i,t}$ at one point in time, there is no aggregation bias from using per capita expenditure E_t/N as the relevant expenditure variable.*

PROOF:

In Section AG of the Appendix. ∎

The key implication of Proposition 2 and Corollary 1 is that the per capita demand can be expressed as a function of the prices, per capita expenditure, as well as an index of inequality in relative marginal utilities. This allows us to empirically identify all the preference parameters from aggregate data except the scale of the function $\mathbf{D}(P)$. Therefore, if the goal is to retrieve preference parameters from aggregate data, then the IA preference class is a natural starting point. The aggregation factor κ and the scale of $\mathbf{D}(P)$ can then be calculated using distributional expenditure data from one period.

IV. IA Preferences: A Simple Parameterization

In this section, we propose a flexible yet simple parameterization of IA preferences that is both suitable for empirical applications and consistent with our

[20] Lewbel (1991) refers to the number of such additive terms as the rank of the demand system.

[21] Our IA class of preferences encompasses the homothetic, the quasi-homothetic, and the PIGL/PIGLOG cases. This can easily be verified from Theorem 1 in Lewbel (1987).

[22] In the proposition, we follow the terminology of Blundell, Pashardes, and Weber (1993), who call κ an aggregation factor.

dynamic multisector framework. To this aim, we focus on the case in (12), which implies aggregate expenditure shares, $\eta_{j,t} \equiv p_{j,t}C_{j,t}/E_t$, of the form

$$(18)\quad \eta_{j,t} = \mathbf{A}_j(P_t)\,p_{j,t}\frac{\mathbf{B}(P_t)}{E_t/N} + \frac{\mathbf{B}_j(P_t)\,p_{j,t}}{\mathbf{B}(P_t)} + \kappa\frac{\mathbf{D}_j(P_t)}{1-\epsilon}p_{j,t}\left(\frac{E_t/N}{\mathbf{B}(P_t)} - \mathbf{A}(P_t)\right)^{1-\epsilon}\frac{\mathbf{B}(P_t)}{E_t/N}.$$

We consider the power form of the class in Proposition 1 since it nests—as we will show further below—both the generalized Stone-Geary and the PIGL preferences.

We parameterize the price function $\mathbf{B}(P_t)$ with a CES aggregator

$$(19)\qquad \mathbf{B}(P_t) = \left(\sum_{j\in J}\omega_j p_{j,t}^{1-\sigma}\right)^{1/(1-\sigma)},$$

where $\sigma > 0$, $\sum_{j\in J}\omega_j = 1$, and $\omega_j \geq 0$. Next, for the function $\mathbf{A}(P_t)$ we choose the form

$$(20)\qquad \mathbf{A}(P_t) = \mathbf{B}(P_t)^{-1}\sum_{j\in J}p_{j,t}\bar{c}_j,$$

where $\bar{c}_j \leq C_{j,t}/N, \forall j \in J$.[23] Finally, the price function $\mathbf{D}(P_t)$ is parameterized by

$$(21)\quad \mathbf{D}(P_t) = \frac{(1-\epsilon)\nu}{\kappa\gamma}\left[\left(\mathbf{B}(P_t)^{-1}\tilde{\mathbf{D}}(P_t)\right)^{\gamma} - 1\right],\quad \tilde{\mathbf{D}}(P_t) = \left(\sum_{j\in J}\theta_j p_{j,t}^{1-\varphi}\right)^{1/(1-\varphi)},$$

where $\nu \geq 0$, $\varphi > 0$, $\sum_{j\in J}\theta_j = 1$, and $\theta_j \geq 0$.[24] We have scaled $\mathbf{D}(P_t)$ with the inverse of the (constant) aggregation factor such that κ cancels in the aggregate expenditure share (18). These functions and parameter restrictions ensure that the expenditure shares add up to unity and that the Slutsky matrix is symmetric. For the intertemporal problem, we additionally restrict $\epsilon < 1$ to ensure that $v(\cdot)$ is strictly increasing and concave in expenditure.

Let $g_{\mathbf{B}}$ and $g_{\tilde{\mathbf{D}}}$ denote the asymptotic gross growth rates of the corresponding price functions in (19) and (21). Then, the asymptotic behavior of the economy is characterized by the following proposition.

PROPOSITION 3: *In our intertemporal framework, the period utility function* (12) *with price functions* (19)–(21) *supports* (i) *an asymptotic balanced growth path and* (ii) *non-negative expenditure shares as* $t \to \infty$ *if* $(g_X/g_{\mathbf{B}})^{\epsilon} > (g_{\tilde{\mathbf{D}}}/g_{\mathbf{B}})^{\gamma}$.

PROOF.
In Section AH of the Appendix. ∎

The proposition shows that, within our framework, the above IA specification is consistent with an asymptotic balanced growth path, and it establishes a sufficient condition under which the expenditure shares remain non-negative. Other flexible

[23] Under additional restrictions outlined below, the parameters $\bar{c}_j$ can be interpreted as subsistence ($\bar{c}_j > 0$) or endowment levels ($\bar{c}_j < 0$) of real sectoral consumption.

[24] When $\sigma \to 1$ or $\varphi \to 1$, then the CES aggregators in (19) and (21) approach the Cobb-Douglas forms $\prod_{j\in J}(p_{j,t})^{\omega_j}$ and $\prod_{j\in J}(p_{j,t})^{\theta_j}$. With $\gamma \to 0$ the function $\mathbf{D}(P)$ approaches $(1-\epsilon)\nu/\kappa\log\left(\mathbf{B}(P)^{-1}\tilde{\mathbf{D}}(P)\right)$.

demand systems, such as the Almost Ideal Demand (AID) and the Quadratic AID (QAID), would violate the asymptotic non-negativity condition in the presence of sustained growth. In general, the condition in part (ii) of the proposition depends on the rates of technical change, but restricting $0 < \gamma \leq \epsilon < 1$ guarantees the condition without further assumptions on these rates.[25] In addition, this simple restriction will allow us to provide a closed form for the direct utility function (see Proposition 4 below), and we will, therefore, impose it in the empirical application of Section V. It is, however, important to stress that the regularity conditions of our preferences do not necessarily require $0 < \gamma \leq \epsilon < 1$, and this restriction could be relaxed when estimating the demand system.

Special Case I: PIGL Preferences.—With $\mathbf{A}(P_t) = 0$, the IA preferences in (12) nest the PIGL class defined in Muellbauer (1975, 1976). The aggregate expenditure shares of PIGL preferences take the form

$$\eta_{j,t} = \frac{\mathbf{B}_j(P_t)\, p_{j,t}}{\mathbf{B}(P_t)} + \kappa \frac{\mathbf{D}_j(P_t)}{1-\epsilon} p_{j,t} \mathbf{B}(P_t)^{\epsilon} \left(E_t/N\right)^{-\epsilon}, \tag{22}$$

where $\kappa = 1/N \int_0^N \left[\left(E_t/N\right)/e_{i,t}\right]^{\epsilon-1} di$. While the PIGL demand system is less flexible than IA, the former has several noteworthy properties. First, the aggregation factor κ is independent of prices and only depends on the parameter ϵ. Second, the parameter ϵ also determines the elasticity of intertemporal substitution (EIS), which is for the PIGL equal to $1/(1-\epsilon)$.[26] In contrast, the EIS of the IA specification equals $1/(1-\epsilon) \cdot \left[1 - \mathbf{A}(P_t)\mathbf{B}(P_t)/e_{i,t}\right]$, and thus varies across households and over time. Finally, when $\mathbf{B}(P)$ is of the Cobb-Douglas form (i.e., when $\sigma \to 1$), the PIGL specification is consistent with an *exact* balanced growth path.[27]

Special Case II: Generalized Stone-Geary Preferences.—The generalized Stone-Geary specification is nested in (12) with price functions (19)–(21) when $\nu = 0$.[28]

This specification is part of the Gorman class. Aggregate expenditure shares are unaffected by the dispersion of $e_{i,t}$ (inequality) and only depend on the per capita expenditure level:

$$\eta_{j,t} = \frac{\omega_j p_{j,t}^{1-\sigma}}{\mathbf{B}\left(P_t\right)^{1-\sigma}} + \left[p_{j,t}\,\bar{c}_j - \frac{\omega_j p_{j,t}^{1-\sigma}}{\mathbf{B}\left(P_t\right)^{1-\sigma}} \sum_{l \in J} p_{l,t}\bar{c}_l\right] \left(E_t/N\right)^{-1}. \tag{24}$$

The parameter σ controls the (asymptotic) price elasticity of demand. The income elasticities are mainly driven by the subsistence levels $\bar{c}_j$. However, with sustained growth, e outgrows all prices, and all terms involving $\bar{c}_j$—and the income elasticities of the shares—converge asymptotically to zero. Note that the key parameter for

[25] With identical rates of technical change across all consumption sectors, only $\epsilon \in (0,1)$ is required to guarantee the asymptotic non-negativity of the shares.

[26] We use the definition of Browning (2005), where the EIS is given by $-v_e(e_{i,t}, P_t)/\left[v_{ee}(e_{i,t}, P_t)\, e_{i,t}\right]$.

[27] In the PIGL case, only the price function $\mathbf{B}(P)$ enters the Euler equation. See Boppart (2014) for such a PIGL specification that permits an exact balanced growth path.

[28] The direct form of the generalized Stone-Geary function is given by

$$u(c) = \frac{1-\epsilon}{\epsilon} \left(\sum_{j \in J} \omega_j^{1/\sigma} \left(c_j - \bar{c}_j\right)^{(\sigma-1)/\sigma}\right)^{\epsilon\sigma/(\sigma-1)}. \tag{23}$$

the EIS, ϵ, drops out of (24) and cannot be identified from the expenditure shares. Finally, as emphasized in the literature, generalized Stone-Geary preferences are only consistent with exact balanced growth for a narrow set of parameterizations (Kongsamut, Rebelo, and Xie 2001; Ngai and Pissarides 2007).

Direct Form of Preferences.—In general, the IA class defined in Proposition 1 does not admit a closed-form solution for the direct utility function. In many cases, however, it can be interpreted as a simple generalization of well-known direct forms. For a simple (homothetic) example, think of utility being a Cobb-Douglas function of two commodity bundles, where each bundle is a potentially distinct CES aggregator of the three sectors. The indirect utility can then be written as $\log(e) - (1-\nu)\log\mathbf{B}(P) - \nu\log\tilde{\mathbf{D}}(P)$, with the CES indices $\mathbf{B}(P)$ and $\tilde{\mathbf{D}}(P)$ as specified above. Whereas, in general, the direct utility function over the three sectors cannot be specified in closed form, an alternative is to write the direct form as a function of six commodities—the three sectoral outputs used in the two bundles—as

$$\frac{(1-\nu)\sigma}{\sigma-1}\log\left(\sum_{j\in J}\omega_j^{1/\sigma}\left(c_j^1\right)^{(\sigma-1)/\sigma}\right) + \frac{\nu\varphi}{\varphi-1}\log\left(\sum_{j\in J}\theta_j^{1/\varphi}\left(c_j^2\right)^{(\varphi-1)/\varphi}\right),$$

where the demand of a particular good in both bundles should be understood as total demand, i.e., $c_j = c_j^1 + c_j^2$. As stated in the next proposition, the same approach works for our parameterized class as well.[29]

PROPOSITION 4: *With* $0 < \gamma \leq \epsilon < 1$, *the direct utility of* (12) *with price functions* (19)–(21) *can be expressed as*

$$(25)\quad \frac{1-\epsilon}{\epsilon}\left(\mathbf{X}_1^{\mathbf{B}}(c^1)\right)^{\epsilon}\left(1 - \frac{\nu\epsilon}{\kappa\gamma}\left[\left(\frac{\frac{\nu\epsilon}{\kappa\gamma}(1-\gamma/\epsilon)}{\mathbf{X}_2^{\mathbf{B}}(c^2)}\right)^{1-\gamma/\epsilon}\left(\frac{\nu/\kappa}{\mathbf{X}_3^{\tilde{\mathbf{D}}}(c^3)}\right)^{\gamma/\epsilon}\right]^{\epsilon/(1-\epsilon)}\right)^{1-\epsilon} + \frac{(1-\epsilon)\nu}{\kappa\gamma},$$

where $c^k = \left(c_A^k, c_M^k, c_S^k\right)$, $k = 1,2,3$ *is a vector, we have* $c_j^k \geq \bar{c}_j^k$, $\forall k, j$, *and* $c_j = \sum_{k=1}^3 c_j^k$, $\bar{c}_j = \sum_{k=1}^3 \bar{c}_j^k$. *Moreover, the generalized Stone-Geary bundles are given by*

$$\mathbf{X}_l^{\mathbf{B}}(c^l) = \left(\sum_{j\in J}\omega_j^{\frac{1}{\sigma}}\left(c_j^l - \bar{c}_j^l\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \quad \text{and} \quad \mathbf{X}_3^{\tilde{\mathbf{D}}}(c^3) = \left(\sum_{j\in J}\theta_j^{\frac{1}{\varphi}}\left(c_j^3 - \bar{c}_j^3\right)^{\frac{\varphi-1}{\varphi}}\right)^{\frac{\varphi}{\varphi-1}},$$

where $l = 1,2$.

PROOF:

In Section AI of the Appendix. ∎

[29] The homothetic example above can indeed be viewed as the limit case of our parameterized class with $\epsilon \to 0$, $\gamma \to 0$, and $\mathbf{A}(P) = 0$.

The proposition establishes that the household problem can be viewed as maximizing (25) over the nine commodities c_A^k, c_M^k, c_S^k, $k = 1,2,3$ subject to the constraint $p_A \sum_{k=1}^{3} c_A^k + p_M \sum_{k=1}^{3} c_M^k + p_S \sum_{k=1}^{3} c_S^k \leq e$. The direct form in (25) is essentially a nested function over three generalized Stone-Geary bundles. Whereas the bundles $\mathbf{X}_2^{\mathbf{B}}$ and $\mathbf{X}_3^{\tilde{\mathbf{D}}}$ enter in a Cobb-Douglas way, their nesting with $\mathbf{X}_1^{\mathbf{B}}$ is slightly more complicated.[30] The restriction $0 < \gamma \leq \epsilon < 1$ ensures the concavity of (25) and that the demand for each commodity is well behaved. In the next section, we estimate the preference parameters under this restriction, such that the direct nine-commodity perspective can indeed be taken.

V. Empirical Application

In this section, we estimate the expenditure system of the parameterized IA preferences and compare its fit with the one of the nested PIGL and generalized Stone-Geary specifications. We impose $0 < \gamma \leq \epsilon < 1$ on the parameters to ensure consistency with Propositions 3 and 4. To identify the preference parameters, we use the variation in the historical data on sectoral prices and nominal final consumption expenditure per capita for the USA, GBR, CAN, and AUS over the period 1900 to 2014.[31] Following Herrendorf, Rogerson, and Valentinyi (2013), we report the feasible generalized nonlinear least squares (FGNLS) estimator with robust standard errors.[32] As the expenditure shares of the three sectors are collinear, we drop one of the sectors (agriculture). The estimation results do not depend on which sector we leave out.

A. *Estimation of Preference Parameters*

We establish our main estimation results using the expenditure shares of final private consumption, but we also show results when including government consumption. Tables 1 and 2 show the main results for the USA, GBR, CAN, and AUS individually, while Table 3 contains the results when we pool the data from all four countries and run the estimation with and without country-sector fixed effects.[33] The columns labeled "IA" show the results for our flexible IA parameterization in (18), those labeled "PIGL" show the results for the PIGL specification in (22), and "SG" stands in for the generalized Stone-Geary specification in (24).

For some specifications, the best model fit occurs when a restricted parameter is at its bound. In such instances, we set the parameter equal to the boundary value

[30] In some cases, when $\gamma = \epsilon$ and the sectors in the bundles are mutually exclusive (e.g., $\omega_S = 1$ and $\theta_S = 0$), (25) gives the closed-form direct utility over three sectors.

[31] Knowing the value of the constant aggregation factor κ is not required for evaluating the prediction of the aggregate expenditure shares and elasticities. However, we also quantify κ using cross-sectional consumption expenditure data for the USA, as explained in Section VD below.

[32] The GNLS estimator accounts for the error correlation between sectoral expenditure shares in a given year. The estimated error correlation matrix is updated iteratively until convergence, which terms the GNLS estimator feasible. If the conditional moments of the errors are stationary, this is equivalent to maximum likelihood estimation with multivariate normal disturbances. A detailed description of the FGNLS estimator, the underlying assumptions, and robust inference is provided in Stata's documentation of the *nlsur* routine.

[33] The parameter estimates not shown in Tables 1–3 are reported in Tables A1–A3 of the Appendix.

Table 1—Estimation, Private Consumption: USA and GBR

	USA			GBR		
	IA (1)	PIGL (2)	SG (3)	IA (4)	PIGL (5)	SG (6)
σ	0.00	0.22	0.13	0.43	0.46	0.47
	$(\cdot)$	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
$\bar{c}_A$	714		714	481		897
	$(\cdot)$		$(\cdot)$	(159)		$(\cdot)$
$\bar{c}_M$	−463		−1,474	446		248
	(315)		(347)	$(\cdot)$		(34)
$\bar{c}_S$	1,289		−3,001	1,292		953
	$(\cdot)$		(705)	$(\cdot)$		(68)
ϵ	0.37	0.71		0.72	0.61	
	(0.02)	(0.03)		(0.04)	(0.01)	
γ	0.37	0.71		0.00	0.00	
	(0.02)	(0.03)		$(\cdot)$	$(\cdot)$	
Observations	104	104	104	97	97	97
AIC	−1,068	−1,003	−1,000	−1,219	−1,186	−1,058
RMSE_A	0.032	0.032	0.033	0.009	0.011	0.019
RMSE_M	0.022	0.026	0.027	0.012	0.012	0.013
RMSE_S	0.017	0.017	0.017	0.015	0.016	0.022

Notes: All variables are based on final private consumption expenditure. Years affected by World War I, World War II, and the Great Depression are excluded. AIC is the Akaike information criterion and RMSE_j is the root mean squared error for sector j. Robust standard errors are reported in parentheses.

Table 2—Estimation, Private Consumption: CAN and AUS

	CAN			AUS		
	IA (1)	PIGL (2)	SG (3)	IA (4)	PIGL (5)	SG (6)
σ	0.00	0.55	0.65	0.00	0.09	0.15
	$(\cdot)$	(0.1)	(0.03)	$(\cdot)$	(0.11)	(0.1)
$\bar{c}_A$	517		721	947		947
	(171)		$(\cdot)$	$(\cdot)$		$(\cdot)$
$\bar{c}_M$	556		−145	−329		−2,180
	$(\cdot)$		(118)	(322)		(681)
$\bar{c}_S$	1,089		−1,229	1,353		−6,891
	$(\cdot)$		(420)	$(\cdot)$		(1,637)
ϵ	0.49	0.34		0.49	0.90	
	(0.06)	(0.04)		(0.25)	(0.02)	
γ	0.49	0.34		0.49	0.90	
	(0.06)	(0.04)		(0.25)	(0.02)	
Observations	77	77	77	63	63	63
AIC	−982	−878	−801	−692	−656	−670
RMSE_A	0.012	0.020	0.029	0.018	0.018	0.017
RMSE_M	0.009	0.011	0.013	0.015	0.019	0.017
RMSE_S	0.018	0.028	0.038	0.018	0.019	0.018

Notes: All variables are based on final private consumption expenditure. Years affected by World War I, World War II, and the Great Depression are excluded. AIC is the Akaike information criterion and RMSE_j is the root mean squared error for sector j. Robust standard errors are reported in parentheses.

TABLE 3—ESTIMATION, PRIVATE CONSUMPTION: POOLED SAMPLE

	Pooled sample (AUS, CAN, GBR, and USA)					
	IA		PIGL		SG	
	(1)	(2)	(3)	(4)	(5)	(6)
σ	0.42	0.00	0.26	0.00	0.17	0.00
	(0.07)	$(\cdot)$	(0.05)	$(\cdot)$	(0.03)	$(\cdot)$
$\bar{c}_A$	714	714			714	714
	$(\cdot)$	$(\cdot)$			$(\cdot)$	$(\cdot)$
$\bar{c}_M$	−117	−989			−1,213	−2,012
	(131)	(478)			(152)	(2,183)
$\bar{c}_S$	1,089	1,089			−2,199	−6,622
	$(\cdot)$	$(\cdot)$			(297)	(8,306)
ϵ	0.51	0.49	0.71	0.70		
	(0.03)	(0.12)	(0.01)	(0.05)		
γ	0.51	0.49	0.71	0.70		
	(0.03)	(0.12)	(0.01)	(0.05)		
Observations	341	341	341	341	341	341
AIC	−3,017	−3,188	−2,971	−3,119	−2,929	−3,093
RMSE_A	0.026	0.027	0.026	0.025	0.028	0.027
RMSE_M	0.027	0.023	0.029	0.025	0.029	0.024
RMSE_S	0.032	0.026	0.035	0.028	0.036	0.029
Fixed effects	No	Yes	No	Yes	No	Yes

Notes: All variables are based on final private consumption expenditure. Years affected by World War I, World War II, and the Great Depression are excluded. AIC is the Akaike information criterion and RMSE_j is the root mean squared error for sector j. Columns 2, 4, and 6 include country-sector fixed effects. Robust standard errors are reported in parentheses.

(with missing standard error) and report standard errors only for the remaining parameters.

Tables 1–3 show that for the IA specification, the parameter ϵ is precisely estimated with values ranging between 0.37 and 0.72. The result that ϵ is significantly below one reinforces our earlier discussion that sustained income effects are important to fit the historical data. The parameter ϵ is also a key determinant of the IA preferences' EIS, which for the USA—evaluated at per capita consumption expenditure—ranges between one and two. This is illustrated in Figure 4, panel A, which shows the predicted EIS of the USA for both the individual and the pooled estimations. Given the slight increase in the predicted EIS, the Euler equation suggests that a roughly constant consumption expenditure growth over time, as observed in the data, is consistent with a moderately decreasing real interest rate. In comparison, the PIGL, which implies a constant EIS of $1/(1-\epsilon)$, predicts an elasticity slightly above three for the USA.

The tables further show that the point estimate of σ, which enters the IA's elasticity of substitution, is positive for GBR (0.43) and in the pooled sample without fixed effects (0.42). In all other cases, the best fit occurs when the parameter is close to zero.[34] Despite these differences, the predicted Allen-Uzawa Elasticities of

[34] In Tables B1 and B2 of the online Appendix, we report the estimation results when the positivity constraint on σ is removed (along with the constraints on φ and γ that are also occasionally binding) for the IA and the PIGL

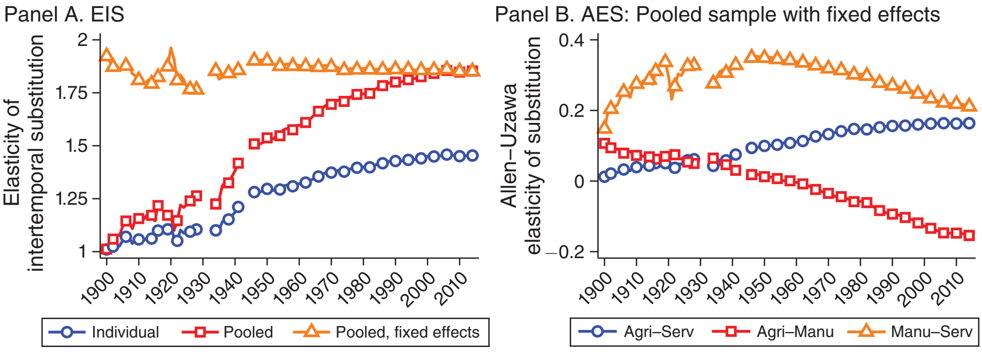

Figure 4. Predicted EIS and AES of the IA Preferences, USA

Notes: Panel A shows the predicted Elasticity of Intertemporal Substitution (EIS) and panel B, the pairwise Allen-Uzawa Elasticity of Substitution (AES) for the USA based on the IA preference estimates. In panel A, circles indicate the prediction of the individual estimation in column 1 of Table 1, squares indicate the prediction of the pooled estimation in column 1 of Table 3, and triangles indicate the prediction of the pooled estimation with fixed effects in column 2 of Table 3. All predictions in panel B are based on the estimates in column 2 of Table 3.

Substitution (AES) are quite similar.[35] In Figure 4, panel B, we plot the AES of the USA based on the pooled estimation with fixed effects. The pairwise AES are systematically estimated below one, indicating the relatively strong complementarity across sectors. In comparison, for the pooled estimation without fixed effects, the predicted AES for the USA range between 0 and 0.5 across sectors and over time. A pairwise AES below one implies that the substitution effect raises the expenditure share of the good with the relative price increase. Figure 4, panel B also highlights the flexibility of the demand system to allow two sectors to be net complements (i.e., a negative AES for agricultural and manufacturing consumption after 1950).[36]

The subsistence or endowment parameters $\bar{c}_j$ remain important to fit the data when using the IA specification. For example, $\bar{c}_S$ is estimated to be positive in all samples, and the best fit occurs when the parameter is at its upper bound, i.e., the minimum per capita service consumption in the data. Note, however, that $\bar{c}_S > 0$ does not directly imply that services are a necessity because the income elasticity of demand also depends on the parameters in $\mathbf{D}(P)$ and on the expenditure level. The flexibility of the income effects is indeed an important feature of the IA preferences: Figure 7, panels C and D below show that service consumption is initially predicted to be a necessity (negative elasticity of the expenditure share) and in later periods a luxury (positive elasticity) for the USA and GBR. Panel C of the same figure shows that US manufacturing consumption is predicted to be a luxury until the 1970s and then turns into a necessity.

specification. This yields a further improvement of the IA's empirical fit, in particular for the USA, GBR, and CAN, but comes at the cost that asymptotically, the Slutsky restrictions are violated.

[35] The AES between good i and j is symmetric and given by $\left[\partial C_{i,t}/\partial p_{j,t} + C_{j,t} \cdot \partial C_{i,t}/\partial E_t\right] \cdot E_t/\left[C_{i,t}C_{j,t}\right]$.

[36] For homothetic CES preferences (i.e., when $\mathbf{A}(P) = \mathbf{D}(P) = 0$), the pairwise AES would be equal to $\sigma > 0$; thus, all sectors must be net substitutes. By contrast, for the considered IA, PIGL, and generalized Stone-Geary specifications, the AES can be negative and generally differs across sector pairs because σ is no longer the sole determinant of the AES.

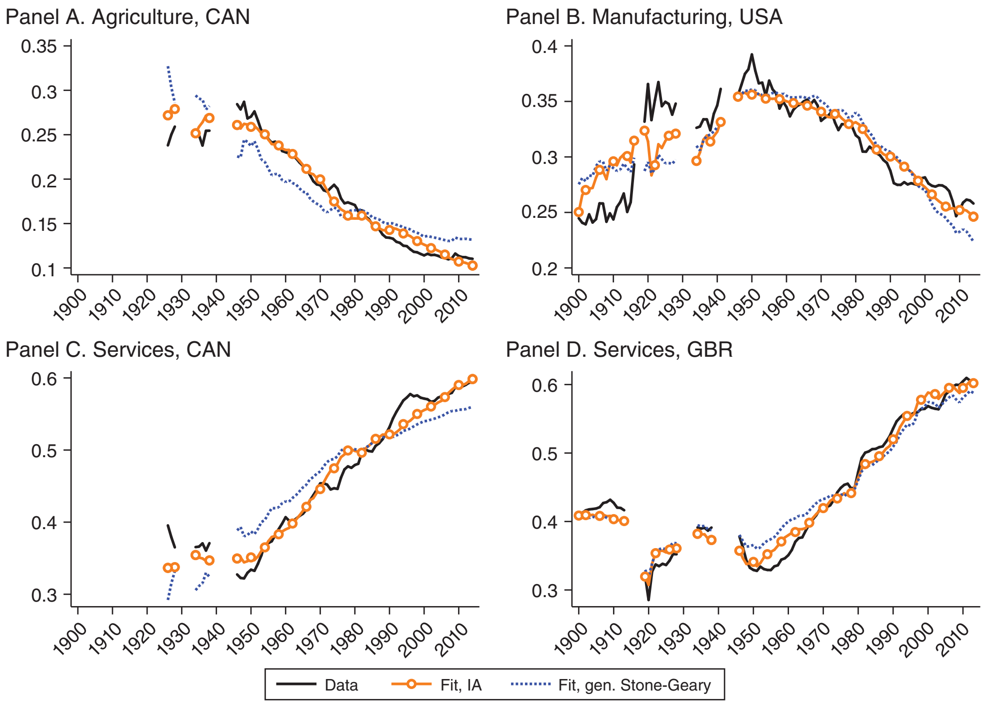

Figure 5. Predicted Final Private Nominal Consumption Expenditure Shares

Notes: The figure plots the predicted final private nominal consumption expenditure shares based on the country-specific estimates in Tables 1 and 2. In each panel, the solid black line shows the data, the orange line with circles indicates the fit of the IA preferences, and the dashed blue line indicates the prediction of the generalized Stone-Geary.

The Akaike Information Criterion (AIC) and Mean Squared Error (RMSE) reported at the bottom of Tables 1–3 indicate that the fit of the historical expenditure shares with IA improves substantially in all samples relative to the generalized Stone-Geary and the PIGL specification.[37] For GBR and CAN—reported in columns 4–6 of Table 1 and columns 1–3 of Table 2, respectively—IA provides a good fit of the agriculture and services shares, for which the difference in the sector-specific RMSE is the largest compared to the Stone-Geary. These differences are confirmed visually in Figure 5, which plots the predicted along with the actual expenditure shares. Finally, while the IA specification with fixed effects naturally yields a better fit than without fixed effects (see Table 3), the differences in the RMSEs between the IA, PIGL, and generalized Stone-Geary remain similar.

Public Consumption Expenditure.—We have repeated the same estimations using the shares of total consumption expenditure, where the service sector also includes

[37] For instance, in Table 1 for the USA, IA achieves the lowest AIC with $-1{,}068$ and the Stone-Geary is merely $\exp\left(\left[-1{,}068-(-1{,}000)\right]/2\right) \approx 0$ times as probable to minimize the information loss.

government expenditure.[38] Tables B3 and B4 of the online Appendix show that the results remain very similar. For the IA preferences, the parameter estimates of ϵ are significantly below one in all samples. Furthermore, the sectoral subsistence consumption is sizeable and for agriculture and services often at its upper bound. Across all samples, the IA specification fits the data better than the generalized Stone-Geary or PIGL, with the exception of the pooled estimation without fixed effects where the fit of the PIGL is similar.

Generalized Stone-Geary.—Due to its prominence in the existing literature, we also briefly discuss the generalized Stone-Geary's estimation results. The second row of Tables 1–3 show that the best fit to the data occurs for all samples when the estimated subsistence level of food is at its upper bound; a $\bar{c}_A$ above food consumption observed in the data would be required to generate strong income effects toward the end of the sample period when per-capita expenditure levels are high. As a consequence, the fall in the expenditure share for agriculture predicted by the generalized Stone-Geary is generally not steep enough to fit the data.[39]

We also find that the point estimate of $\bar{c}_M$ is sizeable and improves the fit of the generalized Stone-Geary specification significantly. For comparison, Table A4 in the Appendix shows the estimation results when $\bar{c}_M$ is restricted to zero—a restriction that is commonly imposed in the literature. Relative to the unrestricted estimations in Tables 1–3, the fit to the data, as measured by the AICs and the RMSEs reported at the bottom of the table, worsens considerably.

B. *Predicted Expenditure Shares*

The predicted nominal expenditure shares of the country-specific estimations in Tables 1–2 are shown in Figure 5. For simplicity, we focus on the IA and generalized Stone-Geary specification and plot the predictions along with the actual shares observed in the data.[40]

Using CAN as an example, panel A of Figure 5 illustrates our earlier result that the generalized Stone-Geary specification underpredicts the sustained decline of the agricultural share because its income effects vanish quickly as per capita expenditure grows. In contrast, IA predicts the decline well because it can generate sustained income effects.[41] Panel B shows that the generalized Stone-Geary underpredicts the increase in the USA's manufacturing sector until 1950, while it overpredicts the decline toward the end of the sample period. IA provides a better fit of the hump

[38] Due to the limited data availability of government expenditure for the USA prior to 1929 (Carter et al. 2006 report numbers for 1902, 1913, 1922, 1927), the number of data points in the USA and the pooled sample reduces by 23 when we consider final total consumption expenditure.

[39] This is most visible for the case of CAN, shown in Figure 5, panel A.

[40] The residuals of the predicted expenditure shares corresponding to Figure 5 are illustrated in Figure B3 of the online Appendix. The predictions for all sectors and countries, and the PIGL specification can be found in Figures B4–B6 of the online Appendix.

[41] From 1950 to 2014 the actual share of agriculture fell by 16.0 percentage points, while the fall predicted by generalized Stone-Geary is merely 10.5. The IA predicts a reduction of 15.6 percentage points.

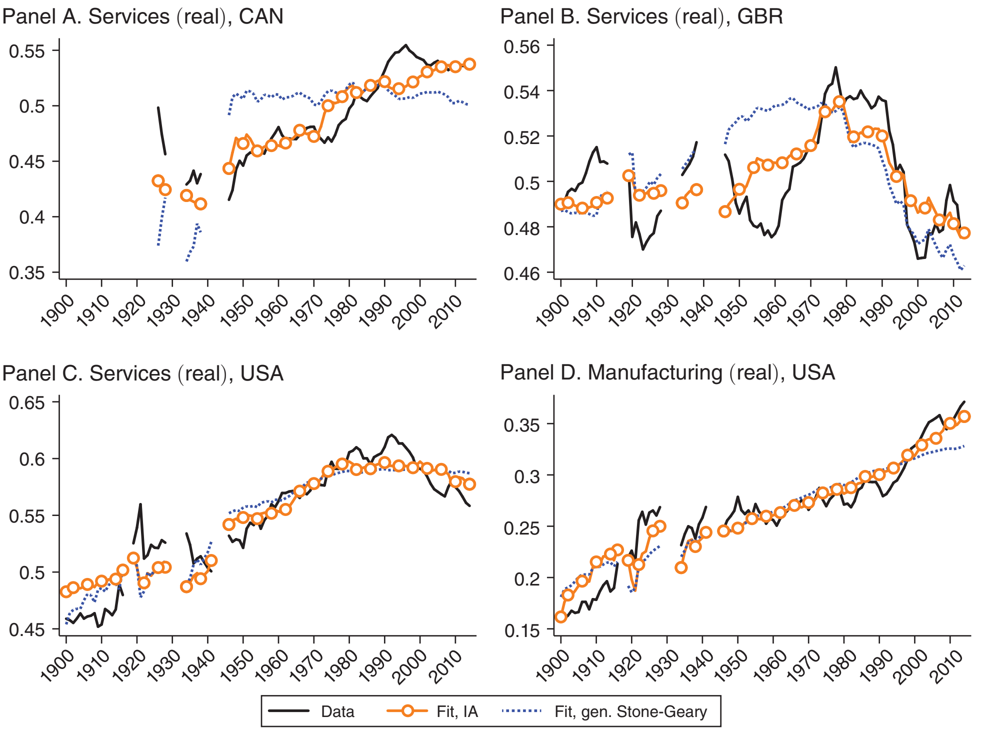

Figure 6. Predicted Final Private Real Consumption Expenditure Shares of Services

Notes: The figure plots the predicted final private real consumption expenditure shares for services based on the country-specific estimates in Tables 1 and 2. In each panel, the solid black line shows the data, the orange line with circles indicates the fit of the IA preferences, and the dashed blue line indicates the prediction of the generalized Stone-Geary.

shape.[42] Panels C and D show for CAN and GBR that the generalized Stone-Geary underpredicts the accelerating increase in the service sector, while IA matches the increase well.[43]

An alternative to the nominal shares is to visualize the data as "real shares," as suggested, for instance, by Herrendorf, Rogerson, and Valentinyi (2014). To this aim, we calculate the predicted and actual real sectoral quantities and express each sector's quantity as a share of the sum of quantities.[44] Figure 6 plots the predicted real expenditure shares based on the estimates in Tables 1 and 2.[45] Panels A–C show that the generalized Stone-Geary struggles to match the pronounced hump shape in the real quantity share of services in CAN, GBR, and the USA, and the fit of IA

[42] The prediction with generalized Stone-Geary is initially too high (26.4 instead of 24.5 percent) and then too low at the end of the sample (24.0 versus 25.8).

[43] The actual service share in CAN increases by 26.0 percentage points between 1950 and 2014. IA predicts an increase of 24.7 percentage points. In GBR, the actual share of services increases by 27.4 percentage points from 1950 to 2013. IA matches this the best and predicts an increase of 26.1 percentage points.

[44] More precisely, the share of real consumption of good j is expressed as a share of the sum of real consumption across all goods, i.e., $c_j/(c_A + c_M + c_S)$, for $j = A, M, S$.

[45] For completeness, we report in Figures B7–B9 of the online Appendix the analog predictions for the remaining sectors, countries, and the PIGL specification.

is generally much better. The difference is starkest in panel A, which shows that in CAN, the real service share increased substantially in the second half of the century and then decreases again, although less than in the USA. IA correctly predicts the strong initial increase and subsequent flattening out, while generalized Stone-Geary yields a relatively constant share in the second half of the century. Furthermore, panel D illustrates that IA predicts the recent rise of real manufacturing in the USA well, while the generalized Stone-Geary underpredicts it.

Overall, the IA preference specification can, due to the more flexible income effects, generate the non-monotonic pattern of structural change the most accurately. We document the role and importance of the flexible income effects in more detail in the next section.

C. *Predicted Income Elasticities*

In this section, we present the predicted income elasticities of the sectoral expenditure shares using the parameter estimates in Tables 1 and 2.[46] For all the considered specifications, the income effects of the sectoral expenditure shares depend on the per capita expenditure level and the sectoral prices, and therefore change over time. When the income elasticity of the expenditure share is positive, the corresponding sector has a luxury character: when income increases, a luxury sector absorbs a larger fraction of total expenditure. Sectors with a negative elasticity of the share have the character of a necessity.

Figure 7 shows the income elasticities of the shares predicted by the IA and generalized Stone-Geary specifications for the USA and GBR.[47] Panels A and B confirm that generalized Stone-Geary predicts income effects that are monotonically converging to zero as the per capita expenditure level increases. This makes it difficult for the specification to match the continued decline in the agricultural sector toward the end of the sample.

For the IA preference specification shown in the lower panels of Figure 7, the predicted income effects are more flexible and sustained. The income elasticity of the agriculture share is substantially below zero over the considered period, which is essential to fit its continued decline. The manufacturing sector starts out as a clear luxury with a high income elasticity. This helps to generate the increasing part of its hump shape. The income elasticity of the manufacturing share then decreases over time and even turns negative for the USA. Thus, in the later years of the USA sample, flexible income effects are crucial to fit the falling expenditure share of manufacturing. Finally, the service expenditure share's income elasticity starts out slightly negative for both countries and is then predicted to be a luxury for most of the later sample period.

[46] The income elasticity is given by $\partial \log(\eta_{j,t}) / \partial \log(E_t/N)$.

[47] The further elasticities are shown in Figures B10–B11 of the online Appendix.

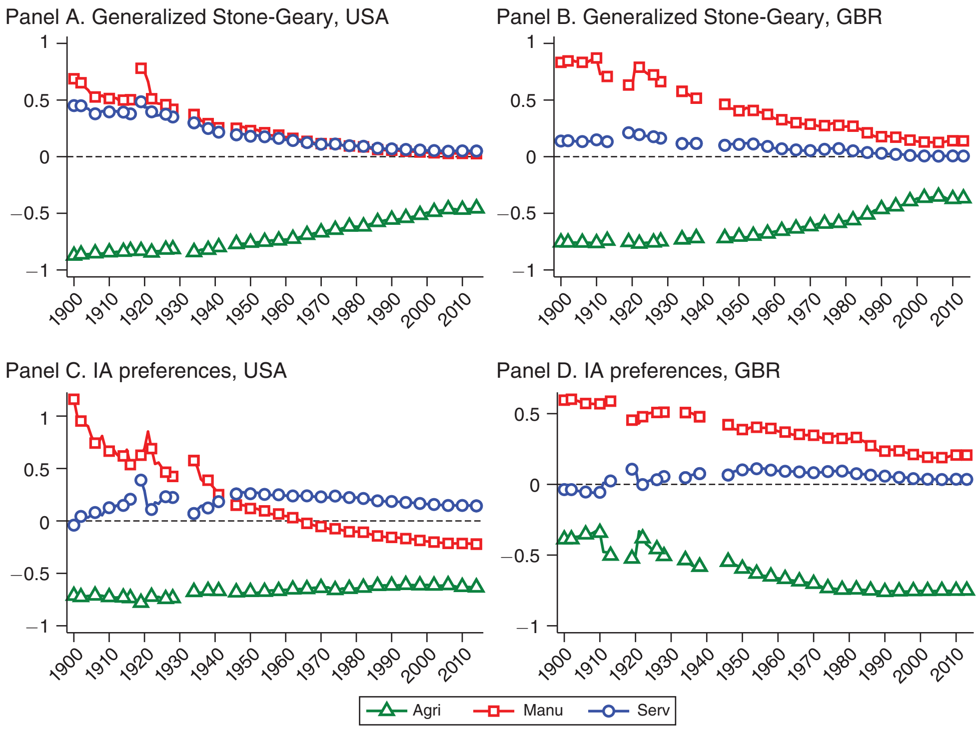

Figure 7. Predicted Income Elasticities of the Expenditure Shares in the USA and GBR

Notes: The figure plots the predicted income elasticities of the sectoral expenditure shares for the USA and GBR based on the estimates in Table 1. Panels A and B show the elasticities predicted by the generalized Stone-Geary specification, and panels C and D show the elasticities predicted by the IA preferences.

D. *Slutsky Restrictions and the Aggregation Factor*

When working with the IA and PIGL specification, parameter restrictions have to ensure the symmetry (SM) and negative semi-definiteness (NSD) of the Slutsky matrix.[48] We enforce the Slutsky restrictions by imposing prohibitive penalties for preference parameters that yield violations of NSD in the standard FGNLS estimation procedure. Thus, all point estimates reported in the tables of the main text and the appendices satisfy SM and NSD pointwise, i.e., when the Slutsky matrix of the household is evaluated at the per capita expenditure and prices observed in each sample.

At the household level, we quantify the constant aggregation factor κ using distributional data from the US Consumer Expenditure Survey for the years 1984–2014.[49] The following iterative procedure is applied to compute κ: (i) we

[48] See Hosoya (2017, Corollary 1), for example. Formally, the Slutsky matrix is given by the Hessian of the household's expenditure function. Since we have already imposed functional forms that guarantee SM, we only need to impose restrictions that ensure the eigenvalues of the Slutsky matrix are non-positive (to check NSD).

[49] We consider average annual consumption expenditures by quintiles of pre-tax income from the US Consumer Expenditure Survey. The data are available for the years 1984–2014 from the US Bureau of Labor Statistics (2022). See online Appendix C.

guess a value for κ; (ii) we estimate the preference parameters for the USA that satisfy the NSD restriction; (iii) based on the point estimates and the distributional data, we compute the updated value of κ as the average value of (17) over the period 1984–2014; (iv) we go back to step (ii) until we reach a fixed point for κ. The resulting κ for the USA is 0.964 for the IA and 0.980 for the PIGL. We then use the US values of κ to estimate the IA and PIGL preference parameters in all other samples.[50]

VI. Relation to Preferences Used in the Literature

In this section, we briefly discuss the relation to other approaches used in the literature and comment on the implications of our findings for applications and estimations of flexible demand systems in dynamic general equilibrium models.

Structural Change and Non-Homothetic Preferences.—In the macroeconomic literature, the papers closest to ours are Buera and Kaboski (2009); Herrendorf, Rogerson, and Valentinyi (2013, 2014); Boppart (2014); and Comin, Lashkari, and Mestieri (2021). We go beyond an analysis of the postwar USA by considering a larger dataset that includes the prewar era for the USA, GBR, CAN, and AUS. The relatively long time period allows us to study the robust regularities documented in Figure 1, including the hump shape in the share of manufacturing. Our conclusions differ from the postwar results in Herrendorf, Rogerson, and Valentinyi (2013) in important ways: a generalized Stone-Geary specification struggles to fit the historical expenditure shares for the majority of countries, including the USA, and income effects are still very important but no longer the single, main force behind structural transformation in final consumption expenditure. We also emphasize the importance of estimating a subsistence level in manufacturing consumption to fit the historical data well, which is typically set to zero in the existing literature.

The result that a generalized Stone-Geary specification is not flexible enough to match the data over a long sample period resembles the finding in Buera and Kaboski (2009). One of our main contributions is to provide a more flexible preference specification that can fit the data. We focus on domestically consumed output, whereas Buera and Kaboski (2009) run the non-homothetic specification over decennial US value-added data from 1870 onward. Isolating the domestic consumption component in the value-added data requires detailed information on import and export, as well as the input-output tables, which are unfortunately not available for the historical data.

As in Boppart (2014) and Comin, Lashkari, and Mestieri (2021), we use a specification that allows for both sustained income and relative price effects in a

[50] When we impose the Slutsky restrictions on the parameter estimates, we check the Slutsky matrix at the household level and need to compute κ, which scales the $\mathbf{D}(P)$ function in the PIGL and the IA period utility. Since $\epsilon \in (0,1)$, higher inequality in expenditures yields lower values of κ and tighter restrictions for the parameters. Thus, using the κ of the USA—which has a relatively high expenditure inequality—for the other countries yields conservative estimates and model predictions.

standard multisector growth framework.[51] However, the IA class of preferences has the additional flexibility to generate—even at constant prices—a non-monotonic relationship between expenditure shares and the expenditure level. While Boppart (2014) considers an economy with two broad sectors for goods and services, we are splitting the goods sector further into agriculture and manufacturing. Comin, Lashkari, and Mestieri (2021) apply the non-homothetic CES specification from Hanoch (1975) in a multisector growth model to study structural change. The IA preferences that we characterize allow us to consistently estimate parameters from historical macroeconomic data without a representative household assumption. The non-homothetic CES specification is not part of the IA class but also allows for sustained income effects. Unlike Boppart (2014) and Comin, Lashkari, and Mestieri (2021), who focus on the postwar period, we provide empirical evidence for the importance of relative price and income effects for the entire twentieth century.[52]

Demand Estimation and Non-Homothetic Preferences.—Our paper is also related to the microeconomic literature on demand system estimation, such as Muellbauer (1975, 1976); Blundell, Pashardes, and Weber (1993); and Banks, Blundell, and Lewbel (1997).

The PIGL class of preferences introduced by Muellbauer (1975, 1976) yields expenditure shares that are quasi-linear in the nominal expenditure level raised to some power (or, in the PIGLOG case, the logarithm of expenditure). Banks, Blundell, and Lewbel (1997) established the QAID system that results from the quadratic generalization of the AID system, which is itself a special case of PIGLOG. Like our IA preferences, the QAID specification allows the expenditure shares to be a non-monotonic function in the expenditure level, as observed for manufacturing in Figure 2. However, there are two important differences from our IA preferences. First, the general QAID specification does not allow for constant aggregation factors as discussed in Blundell, Pashardes, and Weber (1993). In contrast, IA preferences imply a single constant aggregation factor and allow us to identify all preference parameters from aggregate data with cross-sectional information from only one period. Second, the QAID system cannot be used in multisector growth models, because demand becomes negative with sustained growth in per capita expenditure.

Applications and Practical Guidance.—For empirical applications with macroeconomic data, IA preferences are a natural choice because they are the most general class that allows estimating preference parameters without aggregation bias. It is straightforward to extend the framework to more than three sectors if a finer good categorization is required. Researchers who prefer to work with the direct

[51] Kongsamut, Rebelo, and Xie (2001); Ngai and Pissarides (2007); and Foellmi and Zweimüller (2008) shut down either the relative price or the income effect to be consistent with an exact balanced growth path. Like Comin, Lashkari, and Mestieri (2021), we consider specifications consistent with an asymptotic balanced growth path, while Boppart (2014) establishes structural change along an exact balanced growth path.

[52] Leon-Ledesma and Moro (2020) use the PIGL preferences of Boppart (2014) to analyze the US postwar period. Eckert and Peters (2018) apply PIGL preferences to study structural change between the agricultural and the non-agricultural sector in a spatial equilibrium model.

form of preferences, e.g., to state and solve the planner problem, can use function (25) or a special case of it.

IA preferences do not force the income effects to vanish as the income level increases. On the contrary, our preferences allow a good to switch, for example, from being a luxury at low income levels to becoming a necessity at high income levels—even at constant prices. This flexibility of IA preferences, which is not present in the nested PIGL and generalized Stone-Geary forms, is particularly valuable when expenditure shares follow a non-monotonic pattern. Such a pattern—like the hump-shaped manufacturing share—is common in datasets with large variations in income levels, and we illustrate this in our long-run time series and in the cross-sectional microeconomic data. Besides the flexible income effects, IA preferences have flexible elasticities of substitution, where different sectors can be net complements or substitutes.

Since the IA class nests the generalized Stone-Geary as a special case, an applied user can straightforwardly compare the significance of the difference in the fit. In some contexts, with relatively small variations in the income level, a Stone-Geary might indeed suffice. However, in our application, we found that even simple parameterizations of the IA preferences—with a closed-form solution of the direct utility function and the same number of parameters—achieve a substantially better fit than the Stone-Geary specification.[53] When such simple cases do not provide sufficient flexibility to fit the given data, the specification can easily be expanded by considering more general parameterizations within the IA class.

VII. Conclusion

Structural transformation is a stylized fact of modern economic development over the past century, but the existing literature has struggled to provide a theory of consumer demand within a multisector growth model that can fit this long-run reallocation across sectors. We characterize the most general class of intertemporally aggregable preferences that allow for tractable aggregation and consistent estimation of the preference parameters from aggregate data. Based on a novel dataset of historical consumption expenditures of 4 countries over more than 100 years, we show that our preferences provide a better fit for the historical consumption expenditure data than existing theories. One reason is that the standard preferences used in the literature lack the flexibility to fit the non-monotonic pattern in the expenditure shares, which is an essential feature of structural change. Furthermore, our findings have important implications for the external validity of structural transformation in the development process. The observation that the generalized Stone-Geary preferences imply subsistence levels in agriculture that are binding for (not unreasonably)

[53] In the US sample, for instance, the IA specification in (25) with $\gamma = \epsilon$, $\omega_S = 1$, and $\theta_S = 0$ has only seven free parameters and yields the closed-form direct utility function

$$u(c) = \frac{1-\epsilon}{\epsilon}\left(c_S - \bar{c}_S\right)^{\epsilon}\left(1 - \left(\frac{\nu}{\kappa}\right)^{\frac{1}{1-\epsilon}}\left[\sum_{j\in\{A,M\}} \theta_j^{\frac{1}{\varphi}}\left(c_j - \bar{c}_j\right)^{\frac{\varphi-1}{\varphi}}\right]^{\frac{\varphi\epsilon}{(\varphi-1)(\epsilon-1)}}\right)^{1-\epsilon} + \frac{(1-\epsilon)\nu}{\kappa\epsilon},$$

achieving a much lower AIC of $-1{,}059$, compared to $-1{,}000$ for the generalized Stone-Geary.

low income levels limits the ability to apply them to contexts with large variation in incomes across time, countries, or households. We expect that IA preferences avoid this problem and will provide a useful basis for the analysis of structural change in a wide development context. We therefore plan to consider in future work a broader sample of countries. There is an inherent need for a dynamic multisector general equilibrium framework and an empirically robust parameterization of preferences that can be used for welfare analyses of structural change and potential policies, as illustrated by the prominent debate on the effects of deindustrialization (see, for example, Rodrik 2016).

Because of the lack of historical data on home production, we focused exclusively on market expenditure. It would be interesting to extend our analysis and consider how endogenous labor supply and home production interact with the structural change in market expenditure.[54] Finally, another potentially interesting application of IA preferences is the study of the cyclical properties of different sectors.[55]

Appendix A: Lemmata, Proofs, and Additional Tables

A. *Planner Problem*

LEMMA 2: *Let* $\mu^i > 0$ *be the planner's weight on household i. Then, the planner problem in the economy of Section II can be written as*

$$\max_{c_{i,j,t}, k_{j,t}, n_{j,t}} \int_0^N \mu^i v\left(\sum_{j \in J} \tilde{p}_{j,t} c_{i,j,t}, (\tilde{p}_{A,t}, \tilde{p}_{M,t}, \tilde{p}_{S,t})\right) di$$

subject to the resource constraints

$$\int_0^N c_{i,j,t} di \leq k_{j,t}^{\alpha} \left(g_j^t n_{j,t}\right)^{1-\alpha}, \quad \forall j \in J, \tag{A1}$$

$$\sum_{j \in J_+} \left[k_{j,t+1} - (1-\delta) k_{j,t}\right] \leq k_{X,t}^{\alpha} \left(g_X^t n_{X,t}\right)^{1-\alpha}, \tag{A2}$$

$$\sum_{j \in J_+} n_{j,t} \leq n, \tag{A3}$$

for given $k_0 = \sum_{j \in J_+} k_{j,0} > 0, \tilde{p}_{j,t} \equiv \left(g_X / g_j\right)^{(1-\alpha)t} \ \forall j \in J.$

PROOF:

The planner problem is given by

$$\max_{c_{i,j,t}, k_{j,t}, n_{j,t}} \int_0^N \mu^i u\left(c_{i,A,t}, c_{i,M,t}, c_{i,S,t}\right) di \tag{A4}$$

[54] See Moro, Moslehi, and Tanaka (2017) for such an analysis of home production in the postwar period in combination with generalized Stone-Geary preferences.

[55] See Storesletten, Zhao, and Zilibotti (2019) for a unified framework of business cycles and structural change with a nested CES production structure over modern agriculture, subsistence agriculture, and non-agriculture.

subject to (A1)–(A3) and a given $k_0 = \sum_{j \in J_+} k_{j,0} > 0$, $\forall j \in J_+$. Here, $u(\cdot)$ represents the direct utility function defined in (7) and (8). Since $v\left(\sum_{j \in J} \tilde{p}_{j,t} c_{i,j,t}, (\tilde{p}_{A,t}, \tilde{p}_{M,t}, \tilde{p}_{S,t})\right) = u(c_{i,A,t}, c_{i,M,t}, c_{i,S,t})$, we have

$$\partial u(c_{i,A,t}, c_{i,M,t}, c_{i,S,t}) / \partial c_{i,j,t} = v_e\left(\sum_{j \in J} \tilde{p}_{j,t} c_{i,j,t}, (\tilde{p}_{A,t}, \tilde{p}_{M,t}, \tilde{p}_{S,t})\right) \tilde{p}_{j,t}, \quad \forall j \in J.$$

Note that $\tilde{p}_{j,t}$ is the planner's shadow price of producing good j in terms of investments (i.e., the Lagrange multiplier of (A1) divided by the one of (A2)). It is then straightforward to verify that the necessary and sufficient optimality conditions of the problem in (A4) coincide with the ones of the problem in Lemma 2. ∎

B. *Production Side: Equilibrium Conditions*

LEMMA 3: *The capital-labor ratio is equalized across all sectors, i.e.,*

$$\text{(A5)} \qquad \frac{k_{j,t}}{n_{j,t}} = \frac{k_t}{n}, \quad \forall t, \quad \forall j \in J_+.$$

Furthermore, the prices are given by

$$\text{(A6)} \qquad p_{j,t} = g_j^{-(1-\alpha)t} \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{r_t + \delta}{\alpha}\right)^{\alpha} = \left(\frac{g_X}{g_j}\right)^{(1-\alpha)t}, \quad \forall j \in J,$$

where the choice of numéraire $p_{X,t} = 1 = g_X^{-(1-\alpha)t} \left[w_t / (1-\alpha)\right]^{1-\alpha} \left[(r_t + \delta)/\alpha\right]^{\alpha}$ *has been used for the second equality. The equilibrium rental rate and wage rate are given by*

$$\text{(A7)} \qquad r_t + \delta = \alpha \left(\frac{g_X^t n}{k_t}\right)^{1-\alpha}$$

and

$$\text{(A8)} \qquad w_t = (1-\alpha) g_X^t \left(\frac{k_t}{g_X^t n}\right)^{\alpha}.$$

Finally, under optimal production, output can be expressed as

$$\text{(A9)} \qquad y_{j,t} = g_j^{(1-\alpha)t} \left(\frac{k_t}{n}\right)^{\alpha} n_{j,t}, \quad \forall j \in J_+.$$

PROOF:

In each period t, the representative firm in each sector $j \in J_+$ solves

$$\min_{k_{j,t}, n_{j,t}} k_{j,t}(r_t + \delta) + n_{j,t} w_t,$$

subject to an exogenously given output level $\bar{y}_{j,t} = k_{j,t}^{\alpha} (g_j^t n_{j,t})^{1-\alpha}$. The first-order conditions of the firms' problems are

$$\lambda_{j,t} \alpha \bar{y}_{j,t} / k_{j,t} = r_t + \delta$$

and

$$\lambda_{j,t}(1-\alpha)\,\bar{y}_{j,t}/n_{j,t} \;=\; w_t,$$

where $\lambda_{j,t}$ denotes the multiplier attached to the constraint. These first-order conditions directly imply

$$\text{(A10)} \qquad \frac{k_{j,t}}{n_{j,t}} \;=\; \frac{w_t}{r_t+\delta}\cdot\frac{\alpha}{1-\alpha},$$

which, together with (5), implies (A5). Furthermore, this allows us to write output as (A9). Note that $\lambda_{j,t}$ can be interpreted as marginal cost and will be equal to the sectoral price $p_{j,t}$. Solving the first-order conditions for $\lambda_{j,t}$ and combining them with (A10) gives (A6). Finally, with our choice of the numéraire, the first-order conditions of the investment sector imply (A7) and (A8) and establish the lemma. ∎

C. *Proof of Lemma 1*

The Lagrangian of the household problem can be written as

$$\mathcal{L} \;=\; \sum_{t=0}^{\infty}\beta^t v\big(e_{i,t},P_t\big) + \sum_{t=0}^{\infty}\lambda_{i,t}\beta^t\big(a_{i,t}(1+r_t) + w_t n_i - e_{i,t} - a_{i,t+1}\big).$$

The first-order conditions are then given by

$$v_e\big(e_{i,t},P_t\big) \;=\; \lambda_{i,t},$$

$$\lambda_{i,t} \;=\; \lambda_{i,t+1}\beta\big(1+r_{t+1}\big),$$

and

$$a_{i,t}\big(1+r_t\big) + w_t n_i - e_{i,t} \;=\; a_{i,t+1}.$$

The increasing but diminishing marginal utility, i.e., $v_e(\cdot) > 0$ and $v_{ee}(\cdot) < 0$, guarantees an interior solution. Combining the first two first-order conditions then establishes the lemma. ∎

D. *Characterization of a Balanced Growth Path*

LEMMA 4: *Along a balanced growth path, expressed in terms of the investment numéraire, the aggregate capital stock, k_t, aggregate output, $y_t = k_t^{\alpha}\big(g_X^t n\big)^{1-\alpha}$, aggregate expenditure, E_t, and the wage rate, w_t, all grow at constant gross rate g_X, and the interest rate, r_t, is constant.*

PROOF:

Positive capital growth requires positive savings and investments. Hence, along a balanced growth path, we must have $k_t^{\alpha}\big(g_X^t n\big)^{1-\alpha} > E_t$. Then, the resource constraint (10) implies that a constant capital growth rate requires $k_{t+1}/k_t = g_X$. It is then straightforward to see that along this path, output, y_t, and expenditure, E_t, grow at the same gross rate g_X. Finally, (A7) and (A8) imply that the interest rate is constant and that the wage rate grows at gross rate g_X as well. ∎

E. *Proof of Proposition 1*

We start the proof of the proposition with a lemma.

LEMMA 5: *Preferences $\mathcal{U}_{i,0}$ are intertemporally aggregable if and only if there exists a function $z : R \rightarrow R$ such that*

$$v_e(e,P) = z\left(\frac{e}{\mathcal{B}(P)} - \mathcal{A}(P)\right),$$

where $\mathcal{B}(P)$ and $\mathcal{A}(P)$ are functions of prices only.

PROOF:

The marginal utility function must be homogenous of degree minus one, i.e., $v_e(e,P) = x\, v_e(xe, xP)$, for any $x > 0$. Thus, (9) can be expressed as

$$v_e(e_{i,t}, P_t) = v_e(x_{t+1}\, e_{i,t+1}, x_{t+1}\, P_{t+1}), \tag{A11}$$

where $x_{t+1} \equiv \left[\beta(1 + r_{t+1})\right]^{-1}$. Consider a degenerated expenditure distribution with $e_{i,t} = E_t/N$, $\forall i$, where the Euler equation trivially holds at the averages $e_{i,t} = E_t/N$ and $e_{i,t+1} = E_{t+1}/N$. Any mean-preserving cross-sectional distribution can be generated by sequentially redistributing Δ from some household j to another household l. After redistribution, (A11) continues to hold at the average if and only if the marginal impact of current expenditure on future spending is the same for both households, $\partial e_{j,t+1}/\partial(e_{j,t} - \Delta) = \partial e_{l,t+1}/\partial(e_{l,t} + \Delta)$ such that E_{t+1}/N remains unchanged, as well. Since the function $v_e(\cdot)$ is time invariant, this is satisfied if and only if $e_{i,t+1}$ is affine-linearly related to $e_{i,t}$ in the following way:

$$\frac{e_{i,t}}{\mathcal{B}(P_t)} - \mathcal{A}(P_t) = \frac{x_{t+1}\, e_{i,t+1}}{\mathcal{B}(x_{t+1}\, P_{t+1})} - \mathcal{A}(x_{t+1}\, P_{t+1}). \tag{A12}$$

Applying the transformation $z : R \rightarrow R$ to both sides of the above equation yields the individual Euler equation

$$z\left(\frac{e_{i,t}}{\mathcal{B}(P_t)} - \mathcal{A}(P_t)\right) = v_e(e_{i,t}, P_t) = v_e(x_{t+1}\, e_{i,t+1}, x_{t+1}\, P_{t+1}). \tag{A13}$$

This establishes Lemma 5. ∎

Based on Lemma 5, we can now prove Proposition 1. We have

$$v_e(\hat{e}_{i,t}) = x_{t+1}^{-1}\, v_e(\hat{e}_{i,t+1}), \tag{A14}$$

where $\hat{e}_{i,t} \equiv e_{i,t}/\mathcal{B}(P_t) - \mathcal{A}(P_t)$ and $\hat{e}_{i,t+1} \equiv e_{i,t+1}/\mathcal{B}(P_{t+1}) - \mathcal{A}(P_{t+1})$. Using (A13), (A14) can be expressed as

$$z(\hat{e}_{i,t}) = x_{t+1}^{-1} z(\hat{e}_{i,t+1}). \tag{A15}$$

Furthermore, we know from (A12) that $e_{i,t}$ is affine-linearly related to $e_{i,t+1}$, and this property is inherited by $\hat{e}_{i,t}$ and $\hat{e}_{i,t+1}$. Thus, we can write

$$\hat{e}_{i,t+1} = q_0 + q_1 \hat{e}_{i,t},$$

where the terms $q_0 \equiv \left[\mathcal{A}(x_{t+1}P_{t+1})\mathcal{B}(x_{t+1}P_{t+1})\right] / \left[x_{t+1}\mathcal{B}(P_{t+1})\right] - \mathcal{A}(P_{t+1})$ and $q_1 \equiv \mathcal{B}(x_{t+1}P_{t+1}) / \left[x_{t+1}\mathcal{B}(P_{t+1})\right]$ are functions of x_{t+1} and prices in the two periods. Since (A15) needs to hold for all $\hat{e}_{i,t}$, we can differentiate twice with respect to it and arrive at

$$z'(\hat{e}_{i,t}) = x_{t+1}^{-1} z'(\hat{e}_{i,t+1})\, q_1, \tag{A16}$$

$$z''(\hat{e}_{i,t}) = x_{t+1}^{-1} z''(\hat{e}_{i,t+1})\, (q_1)^2. \tag{A17}$$

We can then use equations (A15)–(A17) to get

$$\frac{z''(\hat{e}_{i,t}) z(\hat{e}_{i,t})}{\left[z'(\hat{e}_{i,t})\right]^2} = \frac{z''(\hat{e}_{i,t+1}) z(\hat{e}_{i,t+1})}{\left[z'(\hat{e}_{i,t+1})\right]^2} = Z. \tag{A18}$$

Hence, the second derivative with respect to $\hat{e}_{i,t}$ times the function itself divided by the first derivative squared needs to be equal to a constant (independent of prices, x_{t+1}, and the expenditure level), which we define as Z. We can drop the time index and rewrite (A18) as

$$\frac{z''(\hat{e}_i)}{z'(\hat{e}_i)} = Z\frac{z'(\hat{e}_i)}{z(\hat{e}_i)}.$$

Hence, we have

$$z'(\hat{e}_i) = \mathcal{F}\left[z(\hat{e}_i)\right]^Z, \tag{A19}$$

where $\mathcal{F}$ is a constant. Now we have to distinguish two cases, (i) $Z = 1$ and (ii) $Z \neq 1$.

Case $Z = 1$: The solution to (A19) is

$$z(\hat{e}_i) = \mathcal{G}\exp(\mathcal{F}\hat{e}_i),$$

where $\mathcal{G} > 0$ is some positive constant to ensure positive marginal utility. Hence, Lemma 5 requires that

$$v_e(e_i, P) = \mathcal{G}\exp\left(\mathcal{F}\left(\frac{e_i}{\mathcal{B}(P)} - \mathcal{A}(P)\right)\right). \tag{A20}$$

We then integrate (A20) with respect to e_i to yield the indirect utility function

$$\text{(A21)}\qquad v(e_i,P) = \frac{\mathcal{G}\mathcal{B}(P)}{\mathcal{F}}\exp\left(\mathcal{F}\left(\frac{e_i}{\mathcal{B}(P)} - \mathcal{A}(P)\right)\right) + \mathcal{D}(P),$$

where $\mathcal{D}(P)$ is a new arbitrary function of prices. Since the strict concavity of (A21) in e_i requires that $\mathcal{B}(P)/\mathcal{F} < 0$, a straightforward redefinition of the price functions yields the exponential form of the period utility function in (13).

Case $Z \neq 1$**:** In this case, the solution to (A19) is

$$\text{(A22)}\qquad z(\hat{e}_i) = v_e(\hat{e}_i) = \left[(1-Z)\mathcal{F}\hat{e}_i + \mathcal{G}\right]^{1/(1-Z)},$$

where $\mathcal{F}$ and $\mathcal{G}$ are constants and $(1-Z)\mathcal{F}\hat{e}_i + \mathcal{G} > 0$. When $Z \neq 2$, integration with respect to e_i yields the indirect utility function

$$\text{(A23)}\qquad v(e_i,P) = \frac{\mathcal{B}(P)}{\mathcal{F}(2-Z)}\left[(1-Z)\mathcal{F}\hat{e} + \mathcal{G}\right]^{\frac{2-Z}{1-Z}} + \mathcal{D}(P),$$

where $\mathcal{D}(P)$ is a new arbitrary function of prices. Defining $\epsilon \equiv (2-Z)/(1-Z)$ in (A23) then gives

$$\text{(A24)}\qquad v(e_i,P) = -\frac{\mathcal{B}(P)}{\mathcal{F}}\frac{1-\epsilon}{\epsilon}\left(\frac{1}{1-\epsilon}(-\mathcal{F})\,\hat{e}_i + \mathcal{G}\right)^{\epsilon} + \mathcal{D}(P).$$

Since $v_{ee}(e_i,P) < 0$ requires that $-\mathcal{B}(P)/\mathcal{F} > 0$, we can redefine the price functions in (A24) in an obvious way to yield (12).

Similarly, when $Z = 2$, we can rewrite (A22) as

$$z(\hat{e}_i) = v_e(\hat{e}_i) = \left[-\mathcal{F}\hat{e}_i + \mathcal{G}\right]^{-1},$$

where $\mathcal{F}$ and $\mathcal{G}$ are constants and $-\mathcal{F}\hat{e}_i + \mathcal{G} > 0$. Integration with respect to e_i yields the indirect utility function

$$\text{(A25)}\qquad v(e_i,P) = -\frac{\mathcal{B}(P)}{\mathcal{F}}\log\left[-\mathcal{F}\hat{e}_i + \mathcal{G}\right] + \mathcal{D}(P),$$

where $\mathcal{D}(P)$ is a new function of prices. Since we could add an arbitrary constant to (A25), we can assume without loss of generality that $\mathcal{D}(P) = \log\left(\tilde{\mathcal{D}}(P)\right) > 0$. Redefining the price functions, (A25) can then be expressed as (14).

Finally, the homogeneity restrictions on the price functions are required to ensure the zero homogeneity of the indirect utility functions in prices and nominal expenditure. This concludes the proof of the proposition. ∎

F. *Proof of Proposition 2*

The Marshallian demand (15) follows immediately from applying Roy's identity to (12) and (13). Equation (16) is derived by substituting

$$v_e(e_{i,t}, P_t) = v_e(E_t/N, P_t)\frac{v_e(e_{i,t}, P_t)}{v_e(E_t/N, P_t)}$$

in (15), aggregating over all households, and rearranging terms. Finally, the aggregation factor κ is constant because IA preferences imply that both $v_e(e_{i,t}, P_t)$ and $v_e(E_t/N, P_t)$ grow with the same gross rate $\beta(1 + r_{t+1})$ over time for all households i.

For completeness, as mentioned in the text, we also state here the Marshallian demand system of the remaining IA preference specification (14). Applying Roy's identity to (14) yields the individual demand system

$$\text{(A26)} \quad c_{i,j,t} = \mathbf{A}_j(P_t)\mathbf{B}(P_t) + \frac{\mathbf{B}_j(P_t)}{\mathbf{B}(P_t)} \cdot e_{i,t} + \mathbf{F}_j(P_t)\frac{\log\left(v_e(e_{i,t}, P_t)\frac{\mathbf{B}(P)}{\mathbf{F}(P)}\right)}{v_e(e_{i,t}, P_t)}.$$

In per capita terms, the Marshallian demand of each commodity can be written as

$$\text{(A27)} \quad C_{j,t}/N = \mathbf{A}_j(P_t)\mathbf{B}(P_t) + \frac{\mathbf{B}_j(P_t)}{\mathbf{B}(P_t)} \cdot E_t/N + \mathbf{F}_j(P_t)\frac{\log\left(v_e(E_t/N, P_t)\frac{\mathbf{B}(P)}{\mathbf{F}(P)}\tilde{\kappa}\right)}{v_e(E_t/N, P_t)},$$

where the time-constant aggregation factor is given by

$$\text{(A28)} \quad \tilde{\kappa} \equiv \exp\left(\frac{1}{N}\int_0^N \log\left(\frac{v_e(e_{i,t}, P_t)}{v_e(E_t/N, P_t)}\right)\frac{v_e(E_t/N, P_t)}{v_e(e_{i,t}, P_t)}\, di\right).$$

This completes the proof of Proposition 2. ∎

G. *Proof of Corollary 1*

Since $v_e(e_{i,t}, P_t)$ satisfies the individual Euler equation, the distribution of relative marginal utilities $v_e(E_t/N, P_t)/v_e(e_{i,t}, P_t)$ is constant if and only if preferences are IA. With aggregate data on per capita expenditure and sectoral prices only, (16) allows us to identify all parameters of the IA preferences up to the scale of the function $\mathbf{D}(P)$, and in (A27) all parameters are identified up to a common scalar for $\mathbf{A}(P)$ and $\mathbf{B}(P)^{-1}$. Furthermore, the aggregation factors κ and $\tilde{\kappa}$ only depend on parameters that can be identified with aggregate data alone, as can be seen from (17) and (A28), respectively. Since the aggregation factors do not depend on the unknown scaling, when distributional data for $e_{i,t}$ is available at some point in the data period, then the unknown scales of $\mathbf{D}(P)$ or $\mathbf{A}(P)$ and $\mathbf{B}(P)^{-1}$, respectively, can easily be separated from the corresponding aggregation factors, which are determined by (17) and (A28). ∎

H. *Proof of Proposition 3*

We start the proof by showing part (i) of the proposition. Let $e_t \equiv E_t/N$. Along a balanced growth path (BGP), e_t grows at rate $g_X > 1$, which is strictly greater than any price's growth rate $\left(g_X/g_j\right)^{1-\alpha}$. Thus, along a BGP,

$$\lim_{t\to\infty} p_{j,t}/e_t = 0, \quad \forall j \in J. \tag{A29}$$

Consequently, since $\mathbf{A}(P_t)\left[e_t/\mathbf{B}(P_t)\right]^{-1} = \sum_{j\in J}\left(p_{j,t}/e_t\right)\bar{c}_j$, (A29) implies that along a BGP

$$\lim_{t\to\infty} \mathbf{A}(P_t)\left[e_t/\mathbf{B}(P_t)\right]^{-1} = 0. \tag{A30}$$

Next, the price function $\mathbf{B}(P_t)$ grows at the rate

$$g_{\mathbf{B},t} = \left(\sum_{j\in J}\frac{w_j p_{j,t}^{1-\sigma}}{\sum_{l\in J}\omega_l p_{l,t}^{1-\sigma}}\left(\frac{g_X}{g_j}\right)^{(1-\alpha)(1-\sigma)}\right)^{1/(1-\sigma)}.$$

This growth rate is constant for finite t in the special cases $\sigma \to 1$ or $g_j = g_l$ $\forall j,l \in J$. In all other cases, the growth rate only approaches a constant with $\lim_{t\to\infty} g_{\mathbf{B},t} = \max_{j\in J}\left(g_X/g_j\right)^{1-\alpha}$ if $\sigma < 1$ or $\lim_{t\to\infty} g_{\mathbf{B},t} = \min_{j\in J}\left(g_X/g_j\right)^{1-\alpha}$ if $\sigma > 1$. We define this constant growth rate as $g_{\mathbf{B}} \equiv \lim_{t\to\infty} g_{\mathbf{B},t}$. The Euler equation can be expressed as

$$\left(\frac{1-\mathbf{A}(P_t)\left[e_t/\mathbf{B}(P_t)\right]^{-1}}{1-\mathbf{A}(P_{t+1})\left[e_{t+1}/\mathbf{B}(P_{t+1})\right]^{-1}}\left(e_t/e_{t+1}\right)g_{\mathbf{B},t}\right)^{\epsilon-1} g_{\mathbf{B},t} = \beta\left(1+r_{t+1}\right).$$

Using (A30), it is easy to see that along an asymptotic BGP, the left-hand side of the Euler equation approaches the constant $\left(g_{\mathbf{B}}/g_X\right)^{\epsilon-1} g_{\mathbf{B}}$ and supports a constant interest rate on the right-hand side. In summary, we have shown that the period utility function in (12) with price functions (19)–(21) supports an asymptotic balanced growth path.

Next, we prove part (ii) of the proposition. We can start from the generic form of the expenditure shares in (18) with three additive terms. Given the CES form for $\mathbf{B}(P_t)$, the second term can be expressed as a share $\omega_j p_{j,t}^{1-\sigma}/\left(\sum_{l\in J}\omega_l p_{l,t}^{1-\sigma}\right)$, which is bounded between zero and one. Given (20), the first term can be expressed as

$$\mathbf{A}_j(P_t)\,p_{j,t}\left(\frac{e_t}{\mathbf{B}(P_t)}\right)^{-1} = \frac{p_{j,t}\bar{c}_j}{e_t} - \frac{\omega_j p_{j,t}^{1-\sigma}}{\sum_{l\in J}\omega_l p_{l,t}^{1-\sigma}}\mathbf{A}(P_t)\left(\frac{e_t}{\mathbf{B}(P_t)}\right)^{-1}.$$

Using (A29) and (A30), it is easy to see that $\lim_{t\to\infty} \mathbf{A}_j(P_t)\, p_{j,t} \left[e_t/\mathbf{B}(P_t)\right]^{-1} = 0$. Finally, the third term can be written as

$$\frac{\mathbf{D}_j(P_t)\, p_{j,t}}{v_e(e_t,P_t)\mathbf{B}(P_t)}\left(\frac{e_t}{\mathbf{B}(P_t)}\right)^{-1}\kappa = \nu\frac{\left[\tilde{\mathbf{D}}(P_t)/\mathbf{B}(P_t)\right]^{\gamma}}{\left[e_t/\mathbf{B}(P_t)\right]^{\epsilon}}\left[\frac{\theta_j p_{j,t}^{1-\varphi}}{\sum_{l\in J}\theta_l p_{l,t}^{1-\varphi}} - \frac{\omega_j p_{j,t}^{1-\sigma}}{\sum_{l\in J}\omega_l p_{l,t}^{1-\sigma}}\right]$$
$$\times\left(1 - \mathbf{A}(P_t)\left[e_t/\mathbf{B}(P_t)\right]^{-1}\right)^{1-\epsilon}.$$

The growth rate of $\tilde{\mathbf{D}}(P_t)$, is a weighted average of the growth rates of goods prices such that $g_{\tilde{\mathbf{D}}} < g_X$. Asymptotically, the term $\left[\tilde{\mathbf{D}}(P_t)/\mathbf{B}(P_t)\right]^{\gamma}/\left[e_t/\mathbf{B}(P_t)\right]^{\epsilon}$ grows at the gross rate $\left(g_{\tilde{\mathbf{D}}}/g_{\mathbf{B}}\right)^{\gamma}/\left(g_X/g_{\mathbf{B}}\right)^{\epsilon}$, which is smaller than one under the condition stated in the proposition. Using (A30), we can therefore conclude that

$$\lim_{t\to\infty}\frac{\mathbf{D}_j(P_t)\, p_{j,t}}{v_e(e_t,P_t)\mathbf{B}(P_t)}\left(\frac{e_t}{\mathbf{B}(P_t)}\right)^{-1}\kappa = 0.$$

In summary, we have shown that $\lim_{t\to\infty}\eta_{j,t} = \omega_j p_{j,t}^{1-\sigma}/\left(\sum_{l\in J}\omega_l p_{l,t}^{1-\sigma}\right) \in [0,1]$. This concludes the proof of Proposition 3. ∎

I. *Proof of Proposition 4*

For the proof, we assume parameter values such that the Slutsky matrix is negative semi-definite and the demands $c = (c_A, c_M, c_S)$ are non-negative. Then, the direct utility function u is implicitly defined by the indirect utility function and the demands, i.e., by the following system of equations:

$$u(c) = v(e,P(c)) = \frac{1-\epsilon}{\epsilon}\left(\frac{e - \sum_{j\in J}p_j(c)\bar{c}_j}{\mathbf{B}(P(c))}\right)^{\epsilon} - \frac{(1-\epsilon)\nu}{\kappa\gamma}\left[\left(\frac{\tilde{\mathbf{D}}(P(c))}{\mathbf{B}(P(c))}\right)^{\gamma} - 1\right],$$

$$c_j = -\frac{\partial v(e,P(c))/\partial p_j(c)}{v_e(e,P(c))}, \qquad \forall j \in J.$$

As the indirect utility function and all Marshallian demands are homogeneous of degree zero in e and all prices, we can normalize e to some positive constant. Then, the three demands define a system in the vector c and the three prices p_A, p_M, and p_S. In general, as this system of three equations cannot explicitly be solved for the prices, there is generally no closed form of the direct utility function (in the three quantities). The crux of Proposition 4, however, is that there exists such a closed form when defined over nine commodities instead. Hence, this proof shows that the utility function in (25) defined over nine commodities yields utility $v(e,P)$ given the same budget and prices.

To this aim, we split each sectoral demand into three commodities $c_j = c_j^1 + c_j^2 + c_j^3$ with equal prices $p_j = p_j^k$, $k = 1,2,3$. We then consider the following indirect utility function $\tilde{v}$ that generates the direct utility function $\tilde{u}$

defined over nine commodities $\tilde{c} = \left(c_A^1, c_A^2, c_A^3, c_M^1, c_M^2, c_M^3, c_S^1, c_S^2, c_S^3\right)$ through the following system of equations:

$$\begin{aligned}
\tilde{u}(\tilde{c}) &= \tilde{v}(e, \left(P^1(\tilde{c}), P^2(\tilde{c}), P^3(\tilde{c})\right) \\
&= \frac{1-\epsilon}{\epsilon}\left(\frac{e - \sum_{j \in J}\sum_{k=1}^{3} p_j^k(\tilde{c})\,\bar{c}_j^k}{\mathbf{B}\left(P^1(\tilde{c})\right)}\right)^{\epsilon} \\
&\quad - \frac{(1-\epsilon)\nu}{\kappa\gamma}\left[\left(\frac{\mathbf{B}\left(P^2(\tilde{c})\right)^{1-\gamma/\epsilon}\tilde{\mathbf{D}}\left(P^3(\tilde{c})\right)^{\gamma/\epsilon}}{\mathbf{B}\left(P^1(\tilde{c})\right)}\right)^{\epsilon} - 1\right],
\end{aligned}$$

$$\text{(A31)} \qquad c_j^k = -\frac{\partial \tilde{v}\left(e, \left(P^1(\tilde{c}), P^2(\tilde{c}), P^3(\tilde{c})\right)\right) / \partial p_j^k(\tilde{c})}{\tilde{v}_e\left(e, \left(P^1(\tilde{c}), P^2(\tilde{c}), P^3(\tilde{c})\right)\right)}, \quad \forall j \in J, k = 1,2,3,$$

$$\text{(A32)} \quad P^k(\tilde{c}) = P(c), \quad k = 1,2,3,$$

where $P^k(\tilde{c}) = \left(p_A^k(\tilde{c}), p_M^k(\tilde{c}), p_S^k(\tilde{c})\right)$ is a three-dimensional subvector of the entire price vector, $\sum_{k=1}^{3}\bar{c}_j^k = \bar{c}_j$, and $\bar{c}_j^k \leq c_j^k$. Here, e can again be normalized to some constant. To ease the notation, we supress the argument $\tilde{c}$ of all prices for the remainder of the proof. Condition (A32) ensures that the direct utility is indeed the same as for the three sector formulation

$$\tilde{v}\left(e, \left(P^1, P^2, P^3\right)\right) = \tilde{v}\left(e, (P, P, P)\right) = v(e, P).$$

We first solve for $\tilde{u}$ and verify in a second step that $\tilde{u}$ is concave in $\tilde{c}$. In the first step, we normalize $e - \sum_{j \in J}\sum_{k=1}^{3} p_j^k \bar{c}_j^k = 1$, such that (A31) yields

$$\text{(A33)} \quad c_j^1 - \bar{c}_j^1 = \frac{\omega_j\left(p_j^1\right)^{-\sigma}}{\sum_{l \in J}\omega_l\left(p_l^1\right)^{1-\sigma}}\left(1 - \frac{\nu\epsilon}{\kappa\gamma}\left[\mathbf{B}\left(P^2\right)^{1-\gamma/\epsilon}\tilde{\mathbf{D}}\left(P^3\right)^{\gamma/\epsilon}\right]^{\epsilon}\right), \quad \forall j \in J,$$

$$\text{(A34)} \quad c_j^2 - \bar{c}_j^2 = \frac{\omega_j\left(p_j^2\right)^{-\sigma}}{\sum_{l \in J}\omega_l\left(p_l^2\right)^{1-\sigma}}(1-\gamma/\epsilon)\frac{\nu\epsilon}{\kappa\gamma}\left[\mathbf{B}\left(P^2\right)^{1-\gamma/\epsilon}\tilde{\mathbf{D}}\left(P^3\right)^{\gamma/\epsilon}\right]^{\epsilon}, \quad \forall j \in J,$$

$$\text{(A35)} \quad c_j^3 - \bar{c}_j^3 = \frac{\theta_j\left(p_j^3\right)^{-\varphi}}{\sum_{l \in J}\theta_l\left(p_l^3\right)^{1-\varphi}}\frac{\nu}{\kappa}\left[\mathbf{B}\left(P^2\right)^{1-\gamma/\epsilon}\tilde{\mathbf{D}}\left(P^3\right)^{\gamma/\epsilon}\right]^{\epsilon}, \quad \forall j \in J.$$

Note that we assume that the $\bar{c}_j^k$ terms are such that all c_j^k are non-negative. As long as $\sum_{k=1}^{3}\bar{c}_j^k = \bar{c}_j$ and $\bar{c}_j^k \leq c_j^k$, this is without loss of generality, and it is feasible because total demand $c_j = \sum_{k=1}^{3} c_j^k$ is non-negative. Equation (A33) and (A34) imply

$\left(\omega_l/\omega_j\right)\left(p_l^k/p_j^k\right)^{1-\sigma} = \left(\omega_l/\omega_j\right)^{1/\sigma}\left[\left(c_l^k - \bar{c}_l^k\right)/\left(c_j^k - \bar{c}_j^k\right)\right]^{(\sigma-1)/\sigma}$ for $k = 1,2$. Similarly, (A35) implies that $\left(\theta_l/\theta_j\right)\left(p_l^3/p_j^3\right)^{1-\varphi} = \left(\theta_l/\theta_j\right)^{1/\varphi}\left[\left(c_l^3 - \bar{c}_l^3\right)/\left(c_j^3 - \bar{c}_j^3\right)\right]^{(\varphi-1)/\varphi}$. Thus, (A33)–(A35) can be rearranged for the commodity prices

$$p_j^1 = \frac{\omega_j^{1/\sigma}\left(c_j^1 - \bar{c}_j^1\right)^{-1/\sigma}}{\sum_{l\in J}\omega_l^{1/\sigma}\left(c_l^1 - \bar{c}_l^1\right)^{(\sigma-1)/\sigma}}\left(1 - \frac{\nu\epsilon}{\kappa\gamma}\left[\mathbf{B}\left(P^2\right)^{1-\gamma/\epsilon}\tilde{\mathbf{D}}\left(P^3\right)^{\gamma/\epsilon}\right]^{\epsilon}\right), \quad \forall j \in J,$$

$$p_j^2 = \frac{\omega_j^{1/\sigma}\left(c_j^2 - \bar{c}_j^2\right)^{-1/\sigma}}{\sum_{l\in J}\omega_l^{1/\sigma}\left(c_l^2 - \bar{c}_l^2\right)^{(\sigma-1)/\sigma}}\left(1 - \gamma/\epsilon\right)\frac{\nu\epsilon}{\kappa\gamma}\left[\mathbf{B}\left(P^2\right)^{1-\gamma/\epsilon}\tilde{\mathbf{D}}\left(P^3\right)^{\gamma/\epsilon}\right]^{\epsilon}, \quad \forall j \in J,$$

$$p_j^3 = \frac{\theta_j^{1/\varphi}\left(c_j^3 - \bar{c}_j^3\right)^{-1/\varphi}}{\sum_{l\in J}\theta_l^{1/\varphi}\left(c_l^3 - \bar{c}_l^3\right)^{(\varphi-1)/\varphi}}\frac{\nu}{\kappa}\left[\mathbf{B}\left(P^2\right)^{1-\gamma/\epsilon}\tilde{\mathbf{D}}\left(P^3\right)^{\gamma/\epsilon}\right]^{\epsilon}, \quad \forall j \in J.$$

We can now use all these equations of the prices to construct $\mathbf{B}\left(P^1\right)$, $\mathbf{B}\left(P^2\right)$, and $\tilde{\mathbf{D}}\left(P^3\right)$ as follows:

$$\text{(A36)} \qquad \mathbf{B}\left(P^1\right) = \left(\sum_{j\in J}\omega_j\left(p_j^1\right)^{1-\sigma}\right)^{1/(1-\sigma)}$$
$$= \left(\mathbf{X}_1^{\mathbf{B}}\left(c^1\right)\right)^{-1}\left(1 - \frac{\nu\epsilon}{\kappa\gamma}\left[\mathbf{B}\left(P^2\right)^{1-\gamma/\epsilon}\tilde{\mathbf{D}}\left(P^3\right)^{\gamma/\epsilon}\right]^{\epsilon}\right),$$

$$\text{(A37)} \qquad \mathbf{B}\left(P^2\right) = \left(\sum_{j\in J}\omega_j\left(p_j^2\right)^{1-\sigma}\right)^{1/(1-\sigma)}$$
$$= \left(\mathbf{X}_2^{\mathbf{B}}\left(c^2\right)\right)^{-1}\left(1 - \gamma/\epsilon\right)\frac{\nu\epsilon}{\kappa\gamma}\left[\mathbf{B}\left(P^2\right)^{1-\gamma/\epsilon}\tilde{\mathbf{D}}\left(P^3\right)^{\gamma/\epsilon}\right]^{\epsilon},$$

$$\text{(A38)} \qquad \tilde{\mathbf{D}}\left(P^3\right) = \left(\sum_{j\in J}\theta_j\left(p_j^3\right)^{1-\varphi}\right)^{1/(1-\varphi)}$$
$$= \left(\mathbf{X}_3^{\tilde{\mathbf{D}}}\left(c^3\right)\right)^{-1}\frac{\nu}{\kappa}\left[\mathbf{B}\left(P^2\right)^{1-\gamma/\epsilon}\tilde{\mathbf{D}}\left(P^3\right)^{\gamma/\epsilon}\right]^{\epsilon},$$

where the generalized Stone-Geary bundles $\mathbf{X}_k^{\mathbf{B}}\left(c^k\right)$ and $\mathbf{X}_3^{\tilde{\mathbf{D}}}\left(c^3\right)$ are defined in Proposition 4. This system admits solving for the price indices in closed form. Equations (A37) and (A38) imply that the Cobb-Douglas aggregate can be expressed as

$$\text{(A39)} \quad \mathbf{B}\left(P^2\right)^{1-\gamma/\epsilon}\tilde{\mathbf{D}}\left(P^3\right)^{\gamma/\epsilon} = \left[\left(\frac{\frac{\nu\epsilon}{\kappa\gamma}\left(1 - \gamma/\epsilon\right)}{\mathbf{X}_2^{\mathbf{B}}\left(c^2\right)}\right)^{1-\gamma/\epsilon}\left(\frac{\nu/\kappa}{\mathbf{X}_3^{\tilde{\mathbf{D}}}\left(c^3\right)}\right)^{\gamma/\epsilon}\right]^{1/(1-\epsilon)}.$$

Finally, under the normalization $e - \sum_{j \in J} \sum_{k=1}^{3} p_j^k \bar{c}_j^k = 1$, the direct utility function can be written as

$$\text{(A40)}\quad \tilde{u}(\tilde{c}) = \frac{1-\epsilon}{\epsilon}\left(\frac{1}{\mathbf{B}(P^1)}\right)^{\epsilon} - \frac{(1-\epsilon)\nu}{\kappa\gamma}\left[\left(\frac{\mathbf{B}(P^2)^{1-\gamma/\epsilon}\tilde{\mathbf{D}}(P^3)^{\gamma/\epsilon}}{\mathbf{B}(P^1)}\right)^{\epsilon} - 1\right]$$

$$= \frac{1-\epsilon}{\epsilon}\left(\frac{1}{\mathbf{B}(P^1)}\right)^{\epsilon}\left(1 - \frac{\nu\epsilon}{\kappa\gamma}\left(\mathbf{B}(P^2)^{1-\gamma/\epsilon}\tilde{\mathbf{D}}(P^3)^{\gamma/\epsilon}\right)^{\epsilon}\right) + \frac{(1-\epsilon)\nu}{\kappa\epsilon}.$$

Substituting (A36) and (A39) in (A40) yields the direct utility function (25) stated in the proposition.

It remains to be verified that $0 < \gamma \leq \epsilon < 1$ ensures that (25) is concave in $\tilde{c}$. First, note that the bundle $\mathbf{X}_1^{\mathbf{B}}(c^1)$ is concave in c^1, since $\sigma > 0$. Similarly,

$$\tilde{\mathbf{X}}(c^2, c^3) \equiv \left(\frac{\mathbf{X}_2^{\mathbf{B}}(x^2)}{\frac{\nu\epsilon}{\kappa\gamma}(1-\gamma/\epsilon)}\right)^{1-\gamma/\epsilon}\left(\frac{\mathbf{X}_3^{\tilde{\mathbf{D}}}(c^3)}{\nu/\kappa}\right)^{\gamma/\epsilon}$$

is concave in c^2 and c^3 since $\sigma, \varphi > 0$ and $0 < \gamma/\epsilon \leq 1$. Next, since $0 < \epsilon < 1$, we can express the direct utility function $\tilde{u}$ as an increasing and concave function h of the concave functions $\mathbf{X}_1^{\mathbf{B}}(c^1)$ and $\tilde{\mathbf{X}}(c^2, c^3)$,

$$\tilde{u}(\tilde{c}) = h\left(\mathbf{X}_1^{\mathbf{B}}(c^1), \tilde{\mathbf{X}}(c^2, c^3)\right)$$

$$= \frac{1-\epsilon}{\epsilon}\left(\mathbf{X}_1^{\mathbf{B}}(c^1)\right)^{\epsilon}\left(1 - \frac{\nu\epsilon}{\kappa\gamma}\left(\tilde{\mathbf{X}}(c^2, c^3)\right)^{-\frac{\epsilon}{1-\epsilon}}\right)^{1-\epsilon}.$$

Taken together, this implies that $\tilde{u}$ is concave in $\tilde{c}$ with $0 < \epsilon < 1$.

In summary, we have shown that if $v(e, P) = \max_{c \geq 0} u(c)$ subject to $\sum_{j \in J} p_j c_j \leq e$ and $0 < \gamma \leq \epsilon < 1$, then $v(e, P) = \max_{\tilde{c} \geq 0} \tilde{u}(\tilde{c})$ subject to $\sum_{j \in J} p_j (c_j^1 + c_j^2 + c_j^3) \leq e$, where $\tilde{u}$ is given by (25). ∎

J. *Additional Tables*

In this section we report the remaining parameter estimates of the IA, PIGL, and generalized Stone-Geary specifications (the continuation of Tables 1–3 in Section VA) for all samples in Tables A1–A3 below. Furthermore, in Table A4, we report the estimation results of the generalized Stone-Geary specification when the manufactured subsistence consumption term is restricted to be zero.

Table A1—Estimation, Remaining Parameters, Private Consumption: USA and GBR

	USA			GBR		
	IA (1)	PIGL (2)	SG (3)	IA (4)	PIGL (5)	SG (6)
ω_A	0.000	0.000	0.047	0.000	0.000	0.086
	$(\cdot)$	$(\cdot)$	(0.003)	$(\cdot)$	$(\cdot)$	(0.004)
ω_M	0.059	0.334	0.322	0.431	0.458	0.390
	(0.025)	(0.004)	(0.003)	(0.004)	(0.008)	(0.003)
ω_S	0.941	0.666	0.632	0.569	0.542	0.525
	(0.025)	(0.004)	(0.004)	(0.004)	(0.008)	(0.005)
θ_A	0.159	0.961		0.895	0.354	
	(0.018)	(0.047)		(0.147)	(0.025)	
θ_M	0.841	0.039		0.033	0.166	
	(0.018)	(0.046)		(0.054)	(0.013)	
θ_S	0.000	0.000		0.072	0.480	
	$(\cdot)$	$(\cdot)$		(0.094)	(0.012)	
φ	1.47	7.32		0.00	0.00	
	(0.29)	(3.33)		$(\cdot)$	$(\cdot)$	
ν	13.4	98.9		82.8	116.5	
	(3.6)	(24.2)		(57.1)	(14.2)	
Observations	104	104	104	97	97	97
AIC	−1,068	−1,003	−1,000	−1,219	−1,186	−1,058

Notes: All variables are based on final private consumption expenditure. Years affected by World War I, World War II, and the Great Depression are excluded. AIC is the Akaike information criterion and RMSE_j is the root mean squared error for sector j. Robust standard errors are reported in parentheses.

Table A2—Estimation, Remaining Parameters, Private Consumption: CAN and AUS

	CAN			AUS		
	IA (1)	PIGL (2)	SG (3)	IA (4)	PIGL (5)	SG (6)
ω_A	0.000	0.000	0.077	0.000	0.000	0.020
	$(\cdot)$	$(\cdot)$	(0.005)	$(\cdot)$	$(\cdot)$	(0.003)
ω_M	0.286	0.225	0.325	0.055	0.315	0.276
	(0.029)	(0.021)	(0.006)	(0.201)	(0.013)	(0.027)
ω_S	0.714	0.775	0.598	0.945	0.685	0.704
	(0.029)	(0.021)	(0.011)	(0.201)	(0.013)	(0.027)
θ_A	0.344	0.445		0.009	0.867	
	(0.066)	(0.026)		(0.064)	(0.06)	
θ_M	0.488	0.555		0.116	0.133	
	(0.065)	(0.026)		(0.574)	(0.06)	
θ_S	0.168	0.000		0.875	0.000	
	(0.026)	$(\cdot)$		(0.638)	$(\cdot)$	
φ	2.01	1.41		0.27	0.00	
	(0.12)	(0.09)		(0.4)	$(\cdot)$	
ν	29.8	7.7		451.3	603.7	
	(16.7)	(2.3)		(2,077)	(97.8)	
Observations	77	77	77	63	63	63
AIC	−982	−878	−801	−692	−656	−670

Notes: All variables are based on final private consumption expenditure. Years affected by World War I, World War II, and the Great Depression are excluded. AIC is the Akaike information criterion and RMSE_j is the root mean squared error for sector j. Robust standard errors are reported in parentheses.

Table A3—Estimation, Remaining Parameters, Private Consumption: Pooled Sample

	Pooled sample (AUS, CAN, GBR, and USA)					
	IA		PIGL		SG	
	(1)	(2)	(3)	(4)	(5)	(6)
ω_A	0.000	0.000	0.000	0.000	0.057	0.020
	$(\cdot)$	$(\cdot)$	$(\cdot)$	$(\cdot)$	(0.002)	(0.023)
ω_M	0.259	0.068	0.377	0.237	0.341	0.206
	(0.027)	(0.061)	(0.006)	(0.018)	(0.004)	(0.011)
ω_S	0.741	0.932	0.623	0.763	0.602	0.774
	(0.027)	(0.061)	(0.006)	(0.018)	(0.005)	(0.026)
θ_A	0.302	0.107	0.634	0.431		
	(0.043)	(0.067)	(0.185)	(0.199)		
θ_M	0.698	0.588	0.082	0.045		
	(0.043)	(0.167)	(0.076)	(0.088)		
θ_S	0.000	0.305	0.284	0.523		
	$(\cdot)$	(0.207)	(0.111)	(0.112)		
φ	0.36	0.15	0.00	0.00		
	(0.15)	(0.18)	$(\cdot)$	$(\cdot)$		
ν	28.2	45.6	163.6	208.4		
	(6.3)	(42.8)	(53.4)	(121.9)		
Observations	341	341	341	341	341	341
AIC	−3,017	−3,188	−2,971	−3,119	−2,929	−3,093
Fixed effects	No	Yes	No	Yes	No	Yes

Notes: All variables are based on final private consumption expenditure. Years affected by World War I, World War II, and the Great Depression are excluded. AIC is the Akaike information criterion and RMSE_j is the root mean squared error for sector j. Columns 2, 4, and 6 include country-sector fixed effects. Robust standard errors are reported in parentheses.

Table A4—Estimation, Private Consumption: Generalized Stone-Geary with $\bar{c}_M = 0$

	USA	GBR	CAN	AUS	Pooled sample	
	(1)	(2)	(3)	(4)	(5)	(6)
σ	0.13	0.37	0.77	0.19	0.34	0.00
	(0.03)	(0.02)	(0.03)	(0.15)	(0.03)	$(\cdot)$
$\bar{c}_A$	714	879	721	947	714	714
	$(\cdot)$	(11)	$(\cdot)$	$(\cdot)$	$(\cdot)$	$(\cdot)$
$\bar{c}_S$	−6	522	−975	−818	80	1,009
	(55)	(49)	(94)	(396)	(76)	(58)
ω_A	0.083	0.077	0.081	0.047	0.095	0.000
	(0.004)	(0.003)	(0.003)	(0.004)	(0.003)	$(\cdot)$
ω_M	0.303	0.389	0.324	0.281	0.330	0.204
	(0.004)	(0.004)	(0.004)	(0.016)	(0.003)	(0.011)
ω_S	0.614	0.535	0.594	0.671	0.575	0.796
	(0.003)	(0.005)	(0.006)	(0.018)	(0.005)	(0.011)
Observations	104	97	77	63	341	341
AIC	−952	−1,040	−802	−635	−2,738	−2,971
RMSE_A	0.042	0.018	0.028	0.021	0.037	0.032
RMSE_M	0.033	0.015	0.016	0.021	0.030	0.025
RMSE_S	0.019	0.021	0.041	0.019	0.040	0.031
Fixed effects	No	No	No	No	No	Yes

Notes: All variables are based on final private consumption expenditure. Years affected by World War I, World War II, and the Great Depression are excluded. AIC is the Akaike information criterion and RMSE_j is the root mean squared error for sector j. Column 6 includes country-sector fixed effects. Robust standard errors are reported in parentheses.

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