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Reconsidering Conventional Explanations of the Inverse Productivity–Size Relationship

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Summary. — The inverse productivity–size relationship is one of the oldest puzzles in development economics. Two conventional explanations for the inverse relationship have emerged in the literature: (i) factor market imperfections that cause cross-sectional variation in household-specific shadow prices and (ii) the omission of soil quality measurements. This study employs precise soil quality measurements at the plot level with multiple plots per household so as to test both conventional explanations simultaneously. Empirical results show that only a small portion of the inverse productivity–size relationship is explained by market imperfections and none of it seems attributable to the omission of soil quality measurements.

Key words — inverse relationship, productivity, market failures, soil characteristics, sub-Saharan Africa, Madagascar

1. INTRODUCTION

The existence of an inverse relationship between farm size and output per unit of land, or productivity, is one of the oldest puzzles of development economics. At the height of Lenin's New Economic Policy, which saw *kulaks* retain the right to market their farm surplus, Chayanov (1926/1986) noted the existence of an inverse farm size–productivity relationship among Russian farms. Similarly, Sen (1962) noted that small Indian agricultural households were much more productive than their larger counterparts in what is perhaps the seminal work on this topic. Since then, the inverse relationship has been observed in Africa (Barrett, 1996; Collier, 1983; Kimhi, 2006), Asia (Akram-Lodhi, 2001; Benjamin & Brandt, 2002; Carter, 1984; Heltberg, 1998; Rios & Shively, 2005), Europe (Alvarez & Arias, 2004) and Latin America (Berry & Cline, 1979), among others. [1]

The inverse relationship is at odds with textbook economic theory, which holds that factor productivity should be equal across farms, otherwise the land market would allow land to be sold or leased from lower marginal productivity to higher marginal productivity households. Similarly, within a farm operated by a single household, factor productivity should be equalized across plots else the household could reallocate inputs to increase output. Although it is in theory possible that the agricultural production technology exhibits non-constant returns to scale, applied microeconomists have found such widespread empirical support for the constant returns to scale hypothesis in developing countries that most researchers no longer even test this hypothesis.

From a policy perspective, one may be tempted to naïvely interpret the existence of the inverse relationship as *prima facie* evidence in favor of land redistribution. If small farms are more productive on average than larger farms, it should be sufficient to redistribute land from the latter to the former in order to increase total agricultural productivity and food availability, while simultaneously reducing asset and income inequality.

The explanations offered thus far for the inverse relationship fall into three categories. The first has to do with imperfect factor markets. Sen (1966) theorized that when the labor market is characterized by surplus labor and there exists a wage gap due to a lower real cost of labor on peasant farms than the wage rate on capitalist farms, small peasant farms will be more productive than large capitalist farms. Similarly, Feder (1985) explained the inverse relationship by pointing out that because hired labor is used more intensively on larger farms than on smaller farms, and because wage laborers are more prone to shirking than family labor given imperfect supervision, larger farms tend to be less productive than smaller farms.

For Barrett (1996), imperfections in the land market and the absence of insurance markets push households who own smaller farms—and thus who are more likely to be net buyers of the staple crop—to over-supply labor on their own farms in

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an effort to avoid being exposed to price fluctuations when buying from market. Meanwhile, households who own larger farms—and thus who are more likely to be net sellers of the staple crop—will under-supply labor so as to reduce their exposure to price fluctuations when selling to market. The net result is that smaller farms are more productive than larger farms due to multiple factor market failures.

The key to these imperfect factor markets stories lies in the observation that there may exist unobservable inter-household variation in the shadow prices of factors of production that lead to different input intensity levels that are correlated with farm size, leading to the inverse farm size–productivity relation. Both Sen's (1966) imperfect labor market explanation and Feder's (1985) moral hazard explanations, however, are rejected by Assunção and Braido (2007), who test them using Indian data.

The second type of explanation supposes that the inverse relationship arises due to an omitted relevant variables problem related to the lack of data on soil quality. If soil quality is (positively) associated with crop output and negatively associated with plot or farm size—perhaps due to increased demand for land, which induces greater partitioning of the most productive soils—but analysts lack precise data on soil quality (e.g., soil nutrients), then the omission of soil quality measurements from estimated production functions can bias the coefficient estimates and give rise to a specious inverse relationship (Benjamin, 1995; Bhalla & Roy, 1988). Bhalla and Roy (1988) show that the inclusion of even coarse soil characteristics in the production function weakens the inverse productivity relationship. But their analysis lacks labor data and thus cannot be more specific in discriminating between the imperfect factor markets and soil quality explanations. For Assunção and Braido (2007), who rule out rural factor market imperfections as well as unobserved heterogeneity between households as the causes of the inverse relationship, there is no doubt that "the content of the inverse relationship is related to unobserved characteristics of the plot rather than the household." However, they lack farm- or plot-specific soils data, as does Benjamin (1995), who posited that omitted soil variables explained the inverse relationship in rural Indonesia.

The third type of explanation also supposes that the inverse relationship arises due to statistical issues. Lamb (2003) explains how measurement error may introduce a spurious inverse relationship between size and productivity. If size is measured with error, this measurement error becomes a component of the error term in the regression of interest. But if that measurement error is negatively correlated with size, then the coefficient on land in the regression of interest is biased downwards, which in the limit may cause one to infer that there is a statistically significant inverse relationship between size and productivity. For example, if survey respondents with smaller plots and farms systematically over-report the size of their farm or plots (perhaps because land is a measure of prestige), one is likely to find a spurious inverse relationship between size and productivity. In this case, the inverse relationship is simply a statistical aberration.

This paper revisits this core question in international development using a unique data set that, for the first time (to the best of our knowledge) includes detailed soil quality measurements on multiple plots for each household so that one can test simultaneously the hypotheses that the inverse farm size–productivity relation is caused by factor market imperfections leading to inter-household variation in shadow prices, unobserved soil quality, both, or neither. Using cross-sectional, plot-level data from Madagascar, we estimate production and yield functions that incorporate as regressors detailed soil quality measurements (i.e., carbon, nitrogen, and potassium percentages; soil pH; as well as silt, sand, and clay content) as well as household-level fixed effects to control for unobserved shadow prices. We find that only a modest share of the inverse productivity–size relationship is explained by apparent factor market imperfections that drive variation in household- or village-specific shadow prices, and that none of the inverse relationship seems attributable to the omission of soil quality measurements. These results highlight the possibility that measurement error or intra-household allocative inefficiency cause most of the inverse relationship observed in our data.

The rest of this paper is organized as follows: in Section 2, we discuss the data and present descriptive statistics. Section 3 presents the empirical framework and discusses our identification strategy along with the two approaches used in testing our main hypothesis. In Section 4, we present our estimation results and test whether the inverse relationship arises due to the omission of soil quality measurements, unobserved household-specific shadow factor and output prices, or neither of those mainstream explanations of this oft-observed phenomenon. Section 5 concludes.

2. DATA AND DESCRIPTIVE STATISTICS

In order to study the nature and causes of the inverse relationship between size and productivity, the ideal data set would include data on several plots per household, on several households per village, and on several villages. Moreover, this ideal data set would observe these plots at least twice over time and, because of fertility dynamics, would include precise soil quality measurements for every time period. Given such a data set, one could control for unobserved heterogeneity between plots, households, and villages as well as for soil quality. Unfortunately, no such data set exists. We are able, however, to come reasonably close with a cross-sectional data set that includes detailed socioeconomic and soils measurements at plot level.

The data used in this paper were collected as part of a USAID BASIS Collaborative Research Support Program project carried out in Madagascar in 2002. The survey was fielded in 17 villages in two central highland communes [2] and focused specifically on agricultural households. The villages and households were randomly selected; this paper exploits the household- and plot-level information available in the data and focuses on rice. Because rice is the staple crop in Madagascar in general and in these central highlands areas in particular, any market imperfection is most likely reflected in the average household's rice crops.

The unique feature of these data is that for a sub-sample of the rice plots, five soil core samples were taken at a depth of 15–20 cm in randomly selected locations within each plot and mixed together to create a composite soil sample. The samples were then sent to the World Agroforestry Centre (ICRAF) soil laboratory in Nairobi, Kenya, for wet chemistry and spectral analysis. A total of 1,176 samples were collected from rice plots belonging to 300 households. All 1,176 soil samples went through spectral analysis, and 234 went through wet chemistry analysis. The latter method allowed precise measurement of the carbon, nitrogen, potassium, clay, silt, and sand content as well as the soil pH of each plot. The results of the wet chemistry analysis (i.e., the imputed values for carbon, nitrogen, potassium, clay, silt, and sand content as well as soil pH for 234 plots) were then used as dependent variables in imputing regressions that used principal components scores derived from spectral analysis as their dependent variables.[3] These imputations are shown in Table A1. The appendix provides a somewhat more detailed description of the soils analysis protocol; validation of this method's utility can be found in Shepherd and Walsh (2002).

Table 1 presents descriptive statistics. Yield on the average plot was about 3.7 metric tons of rice per hectare, considerably higher than the national average of about 2.1 metric tons per hectare (Minten & Barrett, 2008). These plots are very small. The average plot covered only 16 ares,[4] producing a little over 600 kg of rice. The average rice plot is worked by 116 person-hours, distributed among four categories of labor (adult, child, hired, and reciprocal help labor), and receives seven hours of draught power, which is usually provided by zebu cattle.

The average household involved in rice production is composed of about seven individuals, half of whom are dependents.[5] Seven percent of household heads are female, and about 11% are single, so that about 4% of single household heads are male and 7% are female. Finally, the average household owned only about a third of a hectare in landholdings.

Turning to the plots themselves, most plots are irrigated, with irrigation being provided by a dam (spring) in 45 (37)% of cases. The average plot was worth roughly US$35 per are, or over $550 in total, to its owner.[6] Carbon, nitrogen, and potassium content on the average plot were 2.4%, 0.2% and 0.2%, respectively. Average soil pH was 5.07, indicating a high level of acidity.[7] Finally, the average plot contained 28.1% clay, 26.4% silt, and 45.2% sand.

These simple descriptive statistics provide the first indication that inter-plot variation in soil quality likely does not play a major explanatory role in the inverse size–productivity relation in these data. These rice plots are relatively uniformly acid, with low soil organic matter (as reflected by carbon content), and high sand and clay content, that is, hardly ideal growing conditions.

Before estimating the regressions discussed in the next section, however, it would be instructive to look at two crude ways of determining whether there indeed exists an inverse size–productivity relationship in our data.

In order to do so, we first computed the coefficient of correlation between yield and cultivated area. This statistic is negative and statistically significant at the 1% level, equal to -0.40 when computed in levels and to -0.58 when measured in logarithms.

The second crude way of checking whether the inverse relationship holds is shown in Figure 1, which presents the result of a non-parametric regression of the logarithm of rice yield on the logarithm of cultivated area using Hastie and Tibshirani's (1990) generalized additive model smoother and the associated 95% confidence band. In this case, the slope of the regression line represents the unconditional elasticity of yield with respect to plot size, which is negative almost everywhere throughout the conditioning domain. By either of these two crude methods, there is ample *prima facie* evidence that the inverse productivity-size relationship holds in these data.

Table 1. *Descriptive statistics (n = 474)*

Variable	Mean	(Std. dev.)
Production variables per plot		
Yield (kg/are)	37.18	(28.00)
Cultivated area (ares)	16.24	(19.02)
Adult labor (person-hours)	37.46	(59.01)
Child labor (person-hours)	6.11	(19.27)
Hired labor (person-hours)	47.27	(74.36)
Help labor (person-hours)	25.73	(59.55)
Draught power (animal-hours)	6.74	(11.31)
Household characteristics		
Household size (individuals)	7.05	(3.10)
Dependency ratio	0.50	(0.21)
Household head female dummy	0.07	(0.25)
Household head single dummy	0.11	(0.31)
Household head single female dummy	0.04	(0.25)
Household head single male dummy	0.07	(0.20)
Total land area owned (ares)	32.36	(35.77)
Plot characteristics		
Plot irrigated by dam dummy	0.45	(0.50)
Plot irrigated by spring dummy	0.37	(0.48)
Plot irrigated by rain dummy	0.15	(0.36)
Plot value (1,000 ariary/are)	51.30	(55.08)
Soil quality measurements		
Imputed carbon percentage	2.41	(1.06)
Imputed nitrogen percentage	0.21	(0.09)
Imputed pH	5.07	(0.30)
Imputed potassium percentage	0.21	(0.07)
Imputed clay percentage	28.17	(3.64)
Imputed silt percentage	26.38	(6.43)
Imputed sand percentage	45.20	(8.07)

3. EMPIRICAL FRAMEWORK

The econometric-theoretic apparatus behind the hypotheses that explain the inverse farm size–productivity relationship is simple. Let the dependent variable Y_{ijk} be the agricultural output on plot i cultivated by household j in village k, and suppose that X_{ijk} is a vector of explanatory variables that are observed by the researcher, that is, inputs and household characteristics that might affect production. Because we are specifically interested in plot size, let A_{ijk} represent cultivated plot area, so that X_{ijk} includes the usual non-land production inputs (e.g., labor and capital), along with village dummies to control for local market prices, climatic conditions, *etc.* Assuming constant returns to scale, we can convert all variables into per hectare terms, with y_{ijk} denoting crop yield (i.e., output per unit area) and x_{ijk} denoting the input application rate per unit area. The production function can then be specified as

$$y_{ijk} = \beta_1' x_{ijk} + \gamma_1 A_{ijk} + \varepsilon_{ijk}, \tag{1}$$

where γ_1 is the parameter of interest that defines the size–yield relationship, β_1 is a vector of coefficients to be estimated, and ε_{ijk} is the error term.

The empirical strategy for establishing which particular explanation for the inverse relationship holds is as follows. First, one would estimate a naïve regression that omits the candidate explanatory variable or variables (e.g., village or household fixed effects for the village- or household-level market imperfections hypotheses; soil quality measurements for the omitted variables hypothesis). Rejecting the null hypothesis $H_0 : \gamma_1 = 0$ in favor of the alternative hypothesis $H_A : \gamma_1 < 0$ would be evidence in favor of the inverse relationship. One would then add the candidate explanatory variable and re-estimate the relationship so as to test the null hypothesis again, looking to see whether (i) the estimate of γ_1 has moved towards zero; (ii) the null can no longer be rejected; or (iii) both. In the literature to date, the fundamental explanations for the inverse size–productivity relationship run as follows.

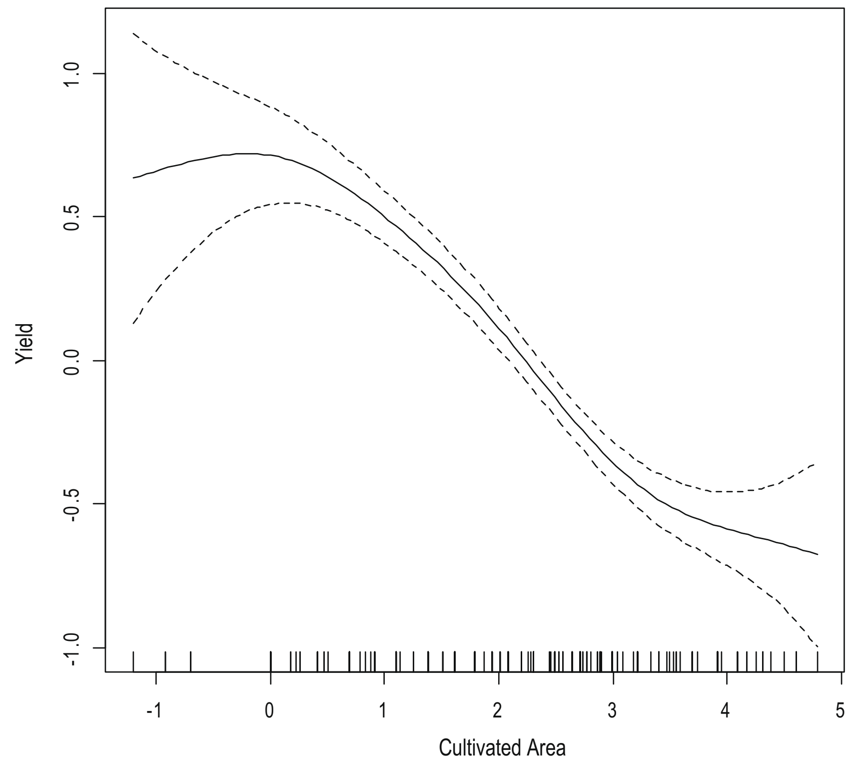

Figure 1. *Nonparametric regression of ln(yield) on ln(cultivated area). Dashed outer lines represent 95% confidence interval.*

The first explanation relies on household-specific market imperfections that cause the shadow prices (i.e., the marginal utility or productivity) of key inputs or outputs to be heterogeneous between households. If land holdings are positively correlated with the unobservable household-specific shadow prices of inputs such as labor or with non-tradable inputs such as supervision, which affects optimal input use rates and thus productivity, yields will be inversely related to land holdings (Feder, 1985). This suggests that one needs to control for the unobserved heterogeneity between households, which may arise from heterogeneous endowments of non-tradables or from heterogeneous shadow prices arising themselves from differences in risk preferences, location, or other attributes between households that affect transactions costs and risk premia (de Janvry, Fafchamps, & Sadoulet, 1991). Under this first hypothesis, the true underlying relationship is

$$y_{ijk} = \beta_2' x_{ijk} + \gamma_2 A_{ijk} + \lambda_{2jk} + v_{ijk}, \tag{2}$$

where λ_{2jk} represents all household-specific not controlled for in X_{ijk} and v_{ijk} is a standard regression error term. If household-specific market failures explain the inverse size–productivity relationship, then controlling for household fixed effects using λ_{2jk} should lead to failure to reject the hypothesis $H_0 : \gamma_2 = 0$ even if the parameter estimates from Eqn. (1) lead to rejection of the null in favor of the one-sided alternate hypothesis. Under the market imperfections line of argument, plot-level variation in yields should be unrelated to plot size once one controls for all household- and, by definition, village-level factors that give rise to market imperfections.

The second explanation found in the literature concerns soil quality Q_{ijk} that is unobserved by the econometrician but known to individual farmers. If plot size is inversely related to soil quality because of more intensive cultivation of better soils, then because farmers adjust the amount of observable inputs used in production in response to soil quality, which is positively correlated with yields and negatively correlated with plot size, a spurious estimated relation between yields and plot size arises due to omitted relevant variables bias. Under this second hypothesis, the true relationship is such that

$$y_{ijk} = \beta_3' x_{ijk} + \gamma_3 A_{ijk} + \phi_3 Q_{ijk} + \eta_{ijk}, \tag{3}$$

where ϕ is the marginal effect of soil quality on productivity and η_{ijk} is the error term. This means that in Eqn. (1), $\varepsilon_{ijk} = \phi_3 Q_{ijk} + v_{ijk}$, and so the usual assumption that $E(A'\varepsilon) = 0$ does not hold since the correlation between soil quality and plot size is likely negative. In that case, the omission of soil quality causes the ordinary least squares (OLS) estimator of the true size–productivity relation to be biased downwards, that is, $E(\hat{\gamma}_1) < \gamma_3$, making it appear that an inverse size–productivity relationship exists when it, in fact, does not. This would hold true as well in the expanded specification that controls for both explanations:

$$y_{ijk} = \beta_4' x_{ijk} + \gamma_4 A_{ijk} + \phi_4 Q_{ijk} + \lambda_{4jk}' + \omega_{ijk}, \tag{4}$$

where ω_{ijk} is the error term and the parameters to be estimated may differ in this expanded specification from the more reductionist Eqn. (1) or the subsequent two equations that each allow for only one non-nested explanation. The nested testing strategy for establishing what best explains the inverse size–productivity relation thus involves proceeding from the simple specification of Eqn. (1) to successively more inclusive specifications to see if and when one ceases rejecting the null hypothesis that $\gamma = 0$. Our goal in this paper is to test both the factor market imperfections and omitted soil quality explanations directly, individually and jointly. The data, described in the preceding section, are uniquely well-suited to pit these two explanations of the oft-observed inverse size–productivity relation against each other directly, for the first time.

We operationalize this empirical strategy using two distinct estimation approaches. The first involves estimating a Cobb–Douglas [8, 9] production function at the plot level by first regressing the natural logarithm of yield on the natural logarithm of production inputs per are (i.e., adult, child, hired, and reciprocal help labor; draught power), [10] household-level controls (i.e., household size, dependency ratio, a dummy for whether the household head is female, and a dummy for whether the household head is single), plot-level characteristics reported by the farmer (i.e., irrigation source dummies), controls for plot value as well as total farm area. [11] For obvious reasons, we label this the naïve regression under the production function approach. One might reasonably worry about the prospective endogeneity of variable inputs such as labor and draught power. We include this production function based approach both because much of the preceding literature on the inverse relationship uses it and this thereby enhances comparability with previous findings, and because it serves as a robustness check on the second, yield-based approach, described below.

We then follow the strategy outlined earlier by next estimating the same production function augmented by including household fixed effects to control for household endowments of non-tradables and market imperfections at the household and village levels. This second model permits direct testing of the market imperfections hypothesis for explaining the inverse productivity–size relation. In a third model aimed at testing the soil quality explanation alone, we drop the household fixed effects and instead include plot-level soil quality variables (i.e., carbon, nitrogen, and potassium percentages; soil pH; and clay, silt, and sand percentages). In our fourth and final model under the production function approach, we include both household fixed effects and the plot-level soil quality measurements, estimating a nested regression specification that simultaneously controls for both market imperfections and soil quality. [12]

Table 2. *Production function approach estimation results (n = 466)*

Variable	(1) Pooled cross-section		(2) Household fixed effects		(3) Soil quality		(4) Household fixed effects and soil quality	
	Coefficient	(Std. err.)	Coefficient	(Std. err.)	Coefficient	(Std. err.)	Coefficient	(Std. err.)
Dependent variable: rice yield (kg/are)								
Production inputs								
Cultivated area	−0.286***	(0.038)	−0.198***	(0.051)	−0.281***	(0.049)	−0.219***	(0.057)
Adult labor per are	−0.007	(0.005)	−0.015	(0.013)	−0.007	(0.005)	−0.012	(0.021)
Child labor per are	−0.004	(0.006)	−0.007	(0.016)	−0.003	(0.006)	−0.002	(0.016)
Hired labor per are	0.014**	(0.006)	0.018	(0.012)	0.014***	(0.005)	0.030***	(0.011)
Help labor per are	−0.001	(0.006)	0.012	(0.013)	0.000	(0.005)	0.013	(0.012)
Draught power per are	0.004	(0.006)	−0.010	(0.010)	0.006	(0.005)	−0.012	(0.011)
Household characteristics								
Household size	0.000	(0.010)			−0.002	(0.009)		
Dependency ratio	−0.102	(0.130)			−0.105	(0.141)		
Single female	−0.027	(0.111)			−0.046	(0.118)		
Single male	0.134	(0.133)			0.129	(0.159)		
Total land area owned	−0.076**	(0.039)			−0.072	0.049		
Plot characteristics								
Irrigated by dam	0.336**	(0.170)	0.199	(0.206)	0.380*	(0.209)	0.570	(0.375)
Irrigated by spring	0.329*	(0.175)	0.207	(0.217)	0.374*	(0.213)	0.556	(0.351)
Irrigated by rain	0.124	(0.179)	−0.038	(0.221)	0.171	(0.217)	0.314	(0.403)
Plot value	0.174***	(0.030)	0.290***	(0.069)	0.168***	(0.032)	0.273***	(0.061)
Soil quality measurements								
Carbon					−1.264	(1.637)	−0.145	(1.947)
Nitrogen					1.512	(1.868)	−0.002	(2.889)
pH					0.562	(7.396)	−25.599*	(15.545)
Potassium					1.096	(1.472)	−7.375**	(3.219)
Clay					0.751	(3.173)	−6.810	(4.415)
Silt					−0.297	(5.875)	8.404	(12.736)
Sand					0.954	(5.823)	6.261	(6.980)
Intercept	−1.715***	(0.371)	−2.986***	(0.703)	−1.748	(1.600)	−3.172***	(0.784)
Number of households	–		286		–		286	
Bootstrap replications	–		–		500		500	
Village fixed effects	Yes		Dropped		Yes		Dropped	
R^2	0.47		0.97		0.48		0.98	
p-Value (all coefficients)	0.00		0.00		0.00		0.00	
p-Value (fixed effects)	–		0.00		–		0.00	
p-Value (soil quality)	–		–		0.67		0.04	

* Indicates statistical significance at the 10% level.
** Indicates statistical significance at the 5% level.
*** Indicates statistical significance at the 1% level.

In the second approach—the yield approach favored by Assunção and Braido (2007)—we first estimate an equally naïve model, treating the sample as a pooled cross-section and regressing the natural logarithm of plot yield on cultivated area, village-level dummy variables, and controls for plot value, total farm area, household characteristics and irrigation dummies. We then estimate a second model aimed at testing the market imperfections hypothesis, now including household fixed effects, which obviously requires dropping household characteristics and the total area cultivated by the household. The third model, as with the production function approach, drops the household fixed effects and adds in plot-specific soil characteristics in order to test the soil quality hypothesis. The fourth model nests the preceding two.

Our empirical strategy thus consists of first testing for the inverse relationship in the most restricted, naïve specification and then seeing whether the inverse relationship disappears once household fixed effects, soil quality measurements, or both are included as controls. Slightly more formally, we test the null hypothesis $H_0 : \gamma = 0$ *versus* the one-sided alternative hypothesis $H_A : \gamma < 0$ in all specifications. The inverse productivity relationship can be said to be the result of omitting precise measures of soil quality if and only if (i) we reject the null in favor of the alternate hypothesis that $\gamma < 0$ when omitting controls for an explanation—household fixed effects in the case of the market imperfections story, soil quality in the case of that explanation—and (ii) we fail to reject the null that $\gamma = 0$ once these controls are included. As a robustness test, we do this following both the production function and yield approaches. These data permit us, for the first time, to use this estimation strategy to nest the imperfect markets and soil quality explanations and to use direct measures of soil quality rather than merely coarse proxy indicators.

4. ESTIMATION RESULTS

Table 2 presents estimation results for four specifications of the production function approach [13]: (i) a naïve pooled cross-sectional specification; (ii) a specification including household fixed effects to control for market imperfections at the household level; (iii) a specification including soil quality measurements; and (iv) a specification including both household fixed effects and soil quality measurements.

The results of the first specification indicate that the inverse relationship holds when we omit both household fixed effects and soil quality measurements. Increasing cultivated area by 1% results on average in a 0.29% decrease in yield. In addition, hired labor has the expected positive sign, and the more land a household owns, the less productive it is on each plot. Finally, plots irrigated by a dam or by a spring are considerably more productive on average than are plots watered by other means, as are higher-valued plots.

In the second column of Table 2 we include household fixed effects to control for possible factor market imperfections that might cause the inverse relationship. Now increasing cultivated area by 1% results on average in a 0.20% decrease in yield, statistically significantly reducing the magnitude of the estimated size–yield relation, but only by roughly one-third. Factor market imperfections seem to only partly explain the inverse size–productivity relationship apparent in these data.

In the third column of Table 2, we include soil quality measurements to test whether the inverse relationship is the result of omitted soil quality variables. Relative to the naive first model, soil quality has no statistically significant impact on the sign, significance, and magnitude of the elasticity of rice yield with respect to plot size. As a further indication

Table 3. *Land value hedonic regressions estimation results (n = 466)*

Variable	(1) Pooled cross-section		(2) Household fixed effects	
	Coefficient	(Bootstrapped std. err.)	Coefficient	(Bootstrapped std. err.)
Dependent variable: log of land value (ariary/are)				
Cultivated area	−0.324***	(0.044)	−0.222***	(0.065)
Plot characteristics				
Irrigated by dam	0.924***	(0.350)	0.781***	(0.290)
Irrigated by spring	1.035***	(0.352)	0.785***	(0.285)
Irrigated by rain	0.762**	(0.359)	0.479	(0.293)
Soil quality measurements				
Carbon	−1.526	(1.852)	0.659	(1.544)
Nitrogen	2.265	(2.329)	−0.918	(2.100)
pH	0.453	(11.797)	4.524	(18.922)
Potassium	−3.870	(2.558)	−2.638	(3.923)
Clay	−3.894	(3.867)	1.120	(3.724)
Silt	−15.543	(11.723)	−5.458	(17.337)
Sand	6.480	(7.289)	10.829	(7.522)
Intercept	14.812***	(2.929)	9.441***	(0.538)
Number of households	–		286	
Bootstrap replications	500		500	
Village fixed effects	Yes		Dropped	
R^2	0.40		0.96	
p-Value (all coefficients)	0.00		0.01	
p-Value (fixed effects)	–		0.00	
p-Value (soil quality)	0.42		0.77	

** Indicates statistical significance at the 5% level.
*** Indicates statistical significance at the 1% level.

that soil quality does not seem to explain the inverse relationship, the p-value on the joint null hypothesis that all the soil quality variables' coefficients equal zero is 0.67. One might be concerned that soil quality is already implicitly captured by the plot value variable. But when we estimate two hedonic regressions of subjective land value, soil quality variables seem to have no statistically significant effect on plot values either (Table 3). Moreover, even if plot value did capture soil quality, the inverse relationship clearly persists even with inclusion of that variable, so, again, soil quality does not appear to explain the observed inverse productivity–size relationship.

Moving to the fourth specification in Table 2, which includes both household fixed effects and soil quality variables, we again find that the estimated inverse relationship is slightly attenuated in magnitude but that increasing cultivated area by 1% still results, on average, in a statistically significant 0.22% decrease in yield. Although both the household fixed effects and the soil quality measurements are jointly statistically significant at the 5% level in this nested specification, they do not adequately explain the apparent inverse relationship.

As a robustness check on our results, Table 4 presents estimation results for the yield approach.[14] Without going through the details of each specification, note that the pattern of results is identical to those reported in Table 2: including household fixed effects (column 2) reduces the estimated coefficient for plot size by about one-third, from −0.27 to −0.18, but soil quality on its own has no explanatory power (column 3), and including both household fixed effects and soil quality measurements (column 4) again weakens the inverse productivity relationship by about one-third but does not make it go away.

Our empirical results are clear. In these data, factor market imperfections appear to explain a modest share of the inverse productivity–size relationship, perhaps one-third of it, but inter-plot variation in soil quality explains essentially none of the inverse relationship. Indeed, inclusion of direct soil quality measures does not even attenuate the estimated inverse size–productivity relationship.

So what could explain the majority residual inverse productivity–size relation? Our results ultimately leave room for two possible explanations. The first explanation is Lamb's (2003) hypothesis that the inverse relationship arises due to measurement error with respect to plot sizes. Testing for this hypothesis, however, would require a good instrument for cultivated area in each plot so that one could credibly control for measurement error in farmer-reported plot size. Unfortunately, our data do not include such an instrument. The second explanation is intra-household allocative inefficiency.[15] For example, households may choose to cultivate more intensively particular plots (e.g., those closer to their residence, or those

Table 4. *Yield approach estimation results (n = 466)*

Variable	(1) Pooled cross-section		(2) Household fixed effects		(3) Soil quality		(4) Household fixed effects and soil quality	
	Coefficient	(Std. err.)	Coefficient	(Std. err.)	Coefficient	(Std. err.)	Coefficient	(Std. err.)
Dependent variable: rice yield (kg/are)								
Cultivated area	−0.271***	(0.038)	−0.176***	(0.046)	−0.265***	(0.048)	−0.187***	(0.052)
Total land area	−0.055	(0.038)			−0.054	(0.047)		
Land value	0.183***	(0.031)	0.303***	(0.069)	0.176***	(0.032)	0.287***	(0.063)
Household characteristics								
Household size	−0.007	(0.009)			−0.008	(0.008)		
Dependency ratio	−0.073	(0.130)			−0.083	(0.144)		
Single female	−0.056	(0.111)			−0.070	(0.119)		
Single male	0.133	(0.134)			0.122	(0.155)		
Plot characteristics								
Irrigated by dam	0.389**	(0.171)	0.228	(0.202)	0.450**	(0.211)	0.545	(0.402)
Irrigated by spring	0.365**	(0.175)	0.250	(0.214)	0.425**	(0.214)	0.541	(0.389)
Irrigated by rain	0.184	(0.180)	0.024	(0.217)	0.249	(0.220)	0.313	(0.431)
Soil quality measurements								
Carbon					−1.361	(1.510)	−0.001	(1.844)
Nitrogen					1.668	(1.781)	−0.007	(2.750)
pH					−1.064	(7.163)	−17.969	(14.459)
Potassium					1.183	(1.412)	−5.528*	(3.035)
Clay					0.293	(3.115)	−5.183	(4.174)
Silt					0.521	(5.751)	4.485	(11.681)
Sand					−0.135	(5.607)	5.261	(7.106)
Intercept	−1.847***	(0.372)	−3.162***	(0.694)	−2.276	(1.552)	−3.328***	(0.819)
Number of households	–		286		–		286	
Bootstrap replications	–		–		500		500	
Village fixed effects	Yes		Dropped		Yes		Dropped	
R^2	0.45		0.97		0.46		0.97	
p-Value (all coefficients)	0.00		0.00		0.00		0.00	
p-Value (fixed effects)	–		0.00		–		0.00	
p-Value (soil quality)	–		–		0.79		0.52	

* Indicates statistical significance at the 10% level.
** Indicates statistical significance at the 5% level.
*** Indicates statistical significance at the 1% level.

they have cultivated for a longer period of time). Accurately testing for this hypothesis, however, would require plot-level longitudinal data, which would allow for plot-level fixed effects. Our data does not allow us to include such plot fixed effects.

5. CONCLUSION

Given the centrality of the inverse productivity–size relation to the agrarian studies and development economics literatures over the years, and the active current debate about the future of small farms in the developing world,[16] it is remarkable that to date no studies have nested the prevailing imperfect markets and omitted soil quality measures explanations in a way that would permit direct comparison of these explanations to see if either, or both, could explain this important feature of smallholder economies.

Using a unique data set that allows pairing the usual household- and plot-level variables used in productivity analyses with precise soil quality measurements and control for household-level unobservables, this paper has tested the hypothesis that the inverse productivity–size relationship results from either factor market imperfections or usually-omitted soil quality measurements. By estimating the yield–plot size relation with and without household fixed effects, we test directly for the possible effect of village- or household-level market imperfections due, for example, to nontradables endowments, risk premia or transactions costs that vary across households and among villages. We find that imperfect markets might explain perhaps one-third of the significant inverse relation in these data, but does not explain most of that effect. And by estimating the yield-plot size relation with and without controls for soil quality—specifically plots-specific measures of soil carbon, nitrogen, and potassium content, soil pH, and clay, silt, and sand shares—we likewise directly test whether omitted soil quality suffices to explain the oft-observed relation, as many previous (including recent) papers suggest. That explanation finds no support whatsoever in these data. These results hold when using either the production function or the yield approach to estimation.

NOTES

1. Some, however, have observed the opposite relationship, that is, large farms being more productive on average than small farms. Hill (1972, 1977) and Kevane (1996) have observed such a relationship in Nigeria and in Sudan. Such cases, however, are the exception rather than the norm.

2. Madagascar's administrative structure is as follows. There are 22 regions, each divided into districts for a total of 116 districts. Within each district are several communes, for a total of 1,548 communes. Finally, there are several villages within each commune.

3. This method, which allows for construction of detailed, plot-specific soil quality variables at high accuracy and reasonable cost, was developed in part because wet chemistry analysis is destructive to the samples on top of being very expensive and time-consuming. The sample size for the wet chemistry analysis is adjusted if necessary so as to ensure a strong correlation between the spectral data and soil characteristics (Shepherd & Walsh, 2002).

4. One hectare = 10,000 square meters. One are = 100 square meters.

5. A household's dependency ratio is obtained by dividing the total number of individuals under 15 and over 65 in the household by the total number of individuals in the household.

6. At the time of writing, US$1 ≈ 1,500 Ariary.

7. Recall that a pH of less than seven denotes an acid, a pH of over 7 denotes a base, and that pure distilled water has a (neutral) pH of 7.

8. For some observations, input values equal zero. Following MaCurdy and Pencavel (1986), we added 0.001 to all observations for all inputs so as not to introduce bias by dropping observations in a non-random fashion.

9. During preliminary work for this paper, we experimented with several flexible functional forms (e.g., quadratic, translog, generalized Leontief, and square root), but the inclusion of interaction terms led to elasticities of an implausible magnitude, especially in the translog case. We thus favor the more restrictive but cleaner and clearer Cobb–Douglas specification.

10. By regressing on inputs per are, we assume constant returns to scale.

11. Plot value captures any plot-specific productivity attributes not accounted for by soil and irrigation measures. Total farm size—the area cultivated by the household across all its plots—captures any scale effects that might exist within the household.

12. Given that the soil quality measurements are imputed, standard errors are bootstrapped whenever these variables appear in a regression.

13. In the interests of brevity, the estimated coefficients for the village fixed effects are not shown, but are available upon request. Moreover, missing data and our focusing on rice plots leave us with information on only 286 of the 300 households from whom soil samples were collected, or 466 plots.

14. Again, the estimated coefficients for the village fixed effects are not shown but are available from the authors upon request.

15. In the abstract, different plots within the household could be operated by different individuals and the resulting intra-household inverse relation could reflect power relations among household members. Udry (1996) documents such intra-household yield variation in Burkina Faso. In the villages surveyed in Madagascar, however, activities are gendered—with women handling rice nurseries and transplanting of seedlings and some weeding activities—but control over plots does not vary significantly within households. So we strongly doubt this explanation holds in the present context.

16. See for example Lipton (2005) or the papers presented at the June 2005 joint IFPRI and ODI workshop on "The Future of Small Farms" at <http://www.ifpri.org/events/seminars/2005/20050626SmallFarms.htm>.

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APPENDIX

Collection of detailed, plot-specific soil quality measures is typically time-consuming and expensive. Most social science data sets therefore omit such measures. This appendix explains how the ICRAF soils laboratory in Nairobi, Kenya, generated plot-specific measures of soil biochemical and physical characteristics from these samples. In order to conserve resources and soil samples, we use recently developed methods due to Shepherd and Walsh (2002), combining traditional wet chemistry analysis of soil samples with spectral analysis that, *via* statistical calibration, significantly leverages analytical resources at a high level of accuracy.

The first step in the analysis involved wet chemistry determination of the exchangeable (i.e., available to the plants) calcium, extractable (i.e., able to be physically quantified by analysis) phosphorus, soil pH, and sand, silt, clay, carbon, and nitrogen content in a 20% random sample of plot-level soil samples collected from the survey rice plots. Samples were air dried, passed through a two millimeter sieve, and then stored in paper bags at room temperature. The extraction method consisted of using 1 molar potassium chloride (KCl) in a 1:10 soil-to-solution ratio. The sample was then analyzed using sodium hydroxide (NaOH) titration to determine the exchangeable acidity. To determine exchangeable calcium and magnesium, the soil was passed through a spectrometer and analyzed by atomic absorption to determine which atoms and how many of them were absorbed by the soil sample. A 0.5 molar sodium carbonate ($NaHCO_3$) plus a 0.01 molar ethylene diamine tetra-acetic acid (EDTA) solution was used to

Table A1. *OLS regressions of soil characteristics established by wet chemistry on PC scores derived from spectral data (n = 234)*

Variable	Carbon (Std.err.)	Nitrogen (Std. err.)	Potassium (Std. err.)	Clay (Std. err.)	Silt (Std. err.)	pH (Std. err.)
Principal component 1	−21.866	−1.610	−0.809	77.237	−17.844	0.411
	(3.630)	(0.308)	(0.708)	(27.731)	(34.451)	(1.809)
Principal component 2	34.244	3.200	−0.058	−147.601	92.245	12.092
	(6.167)	(0.523)	(1.218)	(47.572)	(59.100)	(3.103)
Village fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	0.88	0.89	0.57	0.94	0.90	0.99

extract potassium, which determines exchangeable potassium in the soil (Shepherd & Walsh, 2002). A carbon, nitrogen and sulphur (CNS) analyzer—an automated dry combustion instrument used to measure total carbon, nitrogen, and sulphur in soils (Kowalenko, 2001)—was then used to establish C and N content in the samples. A portion of each sample was treated with H_2SO_4–dichromate oxidation and the percentage of organic carbon in the soil was established by colorimetry. The amount of nitrogen available in the plant was then determined by evaluating ammonium production with 7-d anaerobic incubations at 40 °C (Shepherd & Walsh, 2002).

Unlike wet chemistry analysis, which is expensive (typically $40–$75 per sample), destructive of samples and involves chemical contaminants, spectral analysis is quick, inexpensive, non-destructive and has no potentially dangerous waste products. Spectral analysis was performed on all soil samples using a mid-infrared reflectance (MIR) scanner that recorded reflectance measures at 3,557 different wavelength bands. In this particular data set the soil reflected wavelengths ranging from 420 to 2,260 nm. Not all the reflectance bands explain variation in carbon or nitrogen content (Cozzolino & Morón, 2003; Shepherd & Walsh, 2002), so this method uses partial least squares techniques to linearly combine the 3,557 wavebands into ten principal components. Thus for each soil sample, there were thousands of data points, each representing a particular spectral wavelength. Principal component analysis (PCA) was used to reduce the data to more manageable dimensions. Two principal components suffice to explain the vast majority of the variability in the data.

Plot-level soil characteristics (i.e., carbon; nitrogen; pH; potassium; clay, sand, and silt content) were then predicted using regression-based calibrations based on the PCA scores available for all the soil samples. The calibration method works as follows. For the plot-specific samples subjected to both chemical and spectral analysis, the soil characteristics determined by wet chemistry were regressed, using ordinary least squares, on the PC scores from the spectral analysis of those same samples so as to create a statistical mapping from the spectral measures to the chemical ones for these soils. The first principal component is strongly correlated with soil fertility, particularly with organic carbon and exchangeable bases. The second component represents the remaining organic compounds (see Table A1). Then, similar to the small area estimation method popular in poverty mapping (Elbers, Lanjouw, & Lanjouw, 2003), the resulting parameter estimates were used to predict the soil content of the samples for which there were only PC scores based on spectral measures. The regressions reported in Table A1 include village dummy variables that are not shown but are available upon request from the authors.