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THE RELATION BETWEEN FARM SIZE AND FARM PRODUCTIVITY

The Role of Family Labor, Supervision and Credit Constraints*

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The paper shows that if the performance of hired farmhands is affected by supervision from family members, and if the availability of credit is dependent on the amount of land owned, then a systematic (positive or negative) relationship between per-acre yields and farm size will prevail. A model with no supervision effects on labor productivity would predict that yields are unaffected by farm size. The paper also investigates the relation between land utilization and owned holding size when land rental possibilities are limited. Results are shown to be compatible with various observed patterns in LDCs.

1. Introduction

One of the more frequently cited empirical observations on rural production patterns in less developed countries is the systematic relation between farm size and land productivity. Such a relationship would be expected if the typical agricultural production function is not of constant returns to scale. But, as pointed out by Berry and Cline (1979, pp. 5–7), most evidence tends to suggest that the returns to scale are approximately constant. In the absence of a technological explanation, a number of authors suggested explanations which are based on scale related distortions in factor markets. These explanations are discussed in detail in Berry and Cline (1979, pp. 8–11) and Bhalla (1979, pp. 157–168). The crux of the discussion is that, for a number of reasons, the input prices which different farmers face vary systematically with the size of their holdings, and it is therefore not surprising that input utilization and output/input ratios vary systematically with farm size.

The most frequently cited phenomenon is an inverse relation between farm size and yield per acre [see Deolalikar (1981), Rao and Chotigeat (1981), and

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other studies cited by these authors]. The presence of a dual labor market where smaller farms face cheaper (imputed) labor cost implies higher labor/land ratios on smaller farms and therefore higher pre-acre yields. While the dual labor market hypothesis is the more common explanation for the inverse farm size–productivity relationship [Bhalla (1979, p. 161)], its universal applicability has been challenged [e.g., see Squire (1981, p. 92) on the lack of conclusive evidence]. There is evidence, in fact, that yields may be positively related to farm size or that they do not vary systematically with farm size [see discussion in Berry and Cline (1979, pp. 14, 225)]. These conflicting situations may be reconciled with the existence of price distortions in other factor markets (capital and land) which have countervailing effects.

The purpose of the present paper is to suggest an additional explanation to the variety of observed systematic relationships between per-acre output and farm size. The explanation depends on three intuitively appealing propositions, namely: (a) Hired laborers will be more efficient (i.e., will provide more labor services per unit of time) when subjected to more supervision. (b) Family members, aside from being better motivated than hired laborers, perform a supervisory role with respect to hired labor. (c) The supply of working capital to each farming household is positively related to the amount of land it owns. Once these propositions are accepted, the present analysis shows, output/input ratios (and in particular the output/land ratio) may be systematically related to farm size (either positively or negatively) even when production is subject to constant returns and factor prices are identical for all farmers. These results will not hold if hired labor is not affected by family supervision [propositions (a) and (b)], even if the supply of credit is related to farm size.

The program of the paper is as follows: the next section describes the formal model incorporating propositions (a)–(c). The following section explores the implications of the model for the pattern of input use and yields. The subsequent section develops a number of comparative static results. It is followed by a discussion of the case where a land rental market does not exist. The implications of the model are illustrated by a numerical example. The last section summarizes the results.

2. The model

Consider a region where each farm household consists of F family members capable of conducting farm operations as well as supervising work of hired laborers.[1] The household owns V acres of land, but through renting in or renting out land at the going rental rate R it determines the size of farm it actually operates, denoted by A. Output is assumed to depend on

[1]The issue of child and female labor [Rosenzweig (1980)] is not considered here.

effective labor (L) and land (A). Effective labor is defined as the product of the number of individuals employed and the effort they exert. While family members can be expected to perform farm tasks with maximum effort, say $\bar{e}$, hired laborers' work effort depends on the intensity of the supervision to which they are subjected.[2] It is reasonable to approximate the intensity of supervision by the ratio of household members to operational farm size (F/A), such that for a given household size supervision intensity declines (and the effort exerted by employees declines) with operational farm size. The rationale is that adult members of the households can supervise employees in a given area as they perform farm tasks. The number of employees in the area supervised can vary considerably without affecting the quality of supervision. Denoting effort by e, it is assumed that the marginal returns to supervision intensity are diminishing, i.e.,

$$e=e(F/A), \quad e'>0, \quad e''<0, \quad \lim_{F/A\to\infty} e=\bar{e}. \tag{1}$$

With N hired laborers per operated acre and a total of F household members, the effective labor input is given by[3]

$$L=F\cdot\bar{e}+A\cdot N\cdot e(F/A). \tag{2}$$

Output is determined by a neo-classical production function which depends on effective labor and land,[4]

$$Q=Q(L,A). \tag{3}$$

Assuming constant returns to scale, and utilizing eq. (2) in eq. (3), output per operated acre is given by

$$q=Q[\bar{e}\cdot(F/A)+N\cdot e(F/A);1]=q[\bar{e}\cdot(F/A)+N\cdot e(F/A)], \tag{4}$$

where $q=Q/A$ and $q'>0$, $q''<0$.

[2]See Berry and Cline (1979, p. 6) and Binswanger and Rosenzweig (1983, pp. 33, 34) for a discussion of this issue. The hypothesis that family labor may affect hired labor productivity was not tested empirically to date. There are a few estimates which maintain a distinction between family labor and hired labor [Desai and Mazumdar (1970), Bardhan (1973), Brown and Salkin (1974), Rao and Chotigeat (1981), Deolalikar and Vijverberg (1982)]. Results are mixed, in part due to differing specifications.

[3]One of the specifications of effective labor with which Deolalikar and Vijverberg (1982) experimented is equivalent to eq. (2) under the assumption that F/A is approximately constant across farms. Their estimated coefficients are $\bar{e}=0.758$, $e(F/A)=0.242$.

[4]Capital is not included for simplicity of presentation. The present model assumes implicitly identical capital endowments per acre on all farms. While this abstraction serves to demonstrate the explanatory power of the family supervision role hypothesis, it certainly does not reflect reality. Capital intensity on larger farms tends to be higher, and when this is combined with some other factor market imperfection, systematic relations between farm size and productivity may be observed [Berry and Cline (1979, pp. 10–11)].

It is noteworthy that with *fixed* amounts of family and per-acre hired labor, output per-acre declines when the operational farm size increases ($\partial q/\partial A<0$), since the per-acre input of effective labor declines.

The nature of agricultural production is such that output is forthcoming at the end of an annual or seasonal cultivation period. During this period cash is required to pay for family consumption, hired labor, rented equipment and land and intermediate inputs (fertilizers, pesticides, water, fuel). Working capital is therefore required to facilitate production and to provide for consumption which cannot be deferred. If the capital market were perfect, the model discussed above would be valid, since at the equilibrium interest rate any single farmer could obtain as much credit as is needed. But in reality the supply of credit to any individual farmer is not infinite at a given rate. Rather, at a certain upper limit interest rate, credit rationing and collateral requirements make the supply of working capital facing any given household finite even if a higher interest rate is offered [Smith (1972)]. The supply of credit may thus be a binding constraint, such that the household would like to borrow more than is offered, and cannot obtain more credit even when it is willing to pay a higher interest rate.

A simple but realistic way of introducing a credit market imperfection in the present model is to assume that the supply of credit depends on the amount of land owned by the household because land is the most suitable collateral in the rural economy [Binswanger and Rosenzweig (1983)]. Denote the supply of credit facing any given household by S.[5]

$$S=S(V), \quad S'>0. \tag{5}$$

Denote the wage rate by w, intermediate input costs per acre by c, and cash consumption expenditures per family member during the season by θ.[6] The cash requirements of a family with an operational holding of size A are $w\cdot N\cdot A+c\cdot A+R\cdot(A-V)+\theta\cdot F$, and the working capital constraint faced by the farm is

$$w\cdot N\cdot A+c\cdot A+R\cdot(A-V)+\theta\cdot F\leqq S(V). \tag{6}$$

The farmers' objective is to maximize end of season profits (accounting for interest charges i per dollar borrowed), subject to the working capital

[5]Note that if $V\cdot S'=S$, the imperfection in the capital market does not imply a scale bias in credit supply, i.e., the supply of credit per acre owned is independent of farm size. None of the results reported below depends on a scale bias in credit supply.

[6]It is reasonable to assume that the wage rate is at least equal to consumption requirements, i.e., $w\geqq\theta$.

constraint. Formally,

$$\max_{A,N} \Pi = A\cdot q[\bar{e}\cdot(F/A)+N\cdot e(F/A)]$$

$$-[w\cdot N\cdot A+c\cdot A+R\cdot(A-V)]\cdot(1+i),$$

subject to inequality (6) and $A \geqq 0$, $N \geqq 0$.

Define the Lagrangean function $\Psi \equiv \Pi + \lambda\cdot[S(V)-w\cdot N\cdot A-c\cdot A-R\cdot(A-V)-\theta\cdot F]$, where λ is the shadow price of the credit constraint.

The Kuhn–Tucker conditions for optimization imply[7]

$$\frac{\partial\Psi}{\partial A}=q-q'\cdot[\bar{e}\cdot(F/A)+N\cdot(F/A)\cdot e']$$

$$-(w\cdot N+c+R)\cdot(1+i+\lambda)\leqq 0, \tag{7a}$$

$$\frac{\partial\Psi}{\partial A}\cdot A=0, \tag{7b}$$

$$\frac{1}{A}\cdot\frac{\partial\Psi}{\partial N}=e\cdot q'-w\cdot(1+i+\lambda)\leqq 0, \tag{8a}$$

$$\frac{\partial\Psi}{\partial N}\cdot N=0, \tag{8b}$$

$$\frac{\partial\Psi}{\partial\lambda}=S(v)-w\cdot N\cdot A-c\cdot A-R\cdot(A-V)-\theta\cdot F\geqq 0, \tag{9a}$$

$$\frac{\partial\Psi}{\partial\lambda}\cdot\lambda=0, \tag{9b}$$

$$A\geqq 0,\quad N\geqq 0,\quad \lambda\geqq 0. \tag{10}$$

The economic interpretation of these optimality conditions is straightforward. Considering internal solutions (i.e., $N>0$, $A>0$), eq. (7a) asserts that

[7]For theoretical completeness, the model should also allow the possibility of hiring out household labor to work off the family farm. When the credit constraint is not binding, this omission has no effect since, as will become apparent, all households hire in labor and it would not be optimal to substitute hired labor for high quality family labor. In a subsequent discussion this issue will be reintroduced. It should be noted that due to the seasonality of farming activities, households may be observed both hiring out labor (in the slack season) and hiring in (in peak season). This issue is ignored in the present paper. Second order conditioning for optimum is assumed to hold.

the benefit from an additional operated acre (namely, the per-acre output) should equal the marginal cost of such expansion, which is composed of the per-acre production costs (hired labor, intermediate inputs and opportunity or rental land value) adjusted by the real cost of working capital ($i+\lambda$), and of the cost due to the reduction in per-acre effective labor when the family/(operated farm) ratio declines. Eq. (8a) asserts that the benefit of increasing marginally the per-acre hired labor input (amounting to the effective labor of one hired laborer multiplied by the marginal output of effective labor) should equal the wage rate, adjusted by the real cost of working capital.

3. Implications of the model

Consider first the case where the credit constraint is not binding ($\lambda=0$): solving the first order conditions (7a) and (8a) for the optimal values of A and N and differentiating one obtains

$$\frac{\mathrm{d}A}{\mathrm{d}F}=\frac{A}{F}, \tag{11}$$

$$\frac{\mathrm{d}N}{\mathrm{d}F}=0. \tag{12}$$

Eq. (11) implies that in the absence of binding credit constraints, the elasticity of the optimal operational size with respect to households size is unity, i.e., there is a fixed (operational holding/household size) ratio. The size of owned land does not affect the optimal ratio. This is intuitively expected in a situation of constant returns to scale with perfect rental and capital markets.

Eq. (12) implies that the optimal number of hired laborers per acre is not affected by household size (neither is it affected by the size of the owned holding). Since the earlier results imply that the operational holding is proportional to household size, it follows that the number of hired laborers per acre is identical on all farms, irrespective of the size of the operational holding (and the ratio of family to hired labor declines with operational holding size). A trivial extension of these results is the observation that the level of effective labor per acre is identical on all farms (since the ratio F/A is fixed and N is the same on all farms), assuming all other farm and farmer attributes are identical. It follows therefore that output per acre operated is not affected by the size of the operational farm or by the amount of land owned.

The case where the credit constraint is binding ($\lambda>0$) is of much more interest as it is probably closer to reality. The analysis and the presentation are greatly simplified by assuming that the functions $q(\cdot)$ and $e(\cdot)$ are of fixed elasticity with respect to their arguments, i.e., $(q'/q)\cdot(L/A)\equiv\eta$ and $(e'/e)\cdot(F/A)\equiv\mu$, where η and μ are parameters within the interval (0, 1). The standard treatment of labor in the literature (i.e., assuming that hired labor is not affected by family supervision) is then the special case $\mu=0$ within the present model.

A differentiation of eqs. (7a), (8a) and (9a), under the assumption of an internal solution, yields after some manipulation

$$\frac{\mathrm{d}A}{\mathrm{d}V}=\frac{[(1-\eta-\eta\cdot\mu)\cdot(S'+R)/w]}{[(1-\eta\cdot\mu)\cdot(c+R)]/w-[\mu\cdot(1-\eta)\cdot\bar{e}\cdot F/(e\cdot A)]}. \tag{13}$$

The denominator can be shown to be positive if second order conditions hold. It follows that the sign of eq. (13) is determined by the sign of $(1-\eta-\eta\cdot\mu)$, which is the limit value of total output elasticity with respect to land as the share of family labor tends to zero.[8] When hired laborers' effort is significantly affected by supervision (i.e., μ is large) the term $1-\eta-\eta\cdot\mu$ may be negative, implying low marginal increments to total output if additional land is brought into cultivation. The larger landowner is therefore better off utilizing more of his credit for hiring laborers and employing them on a smaller operational farm.

The percentage of owned area which is operated by the household may decline or increase with owned farm size, as indicated by the following calculation [utilizing eq. (30) and the first order conditions]:

$$\frac{V}{A}\cdot\frac{\mathrm{d}A}{\mathrm{d}V}=(\varepsilon\cdot S+R\cdot V)\Big/\left\{(S+R\cdot V)+F\cdot\left[\frac{(1-\mu-\eta+\eta\cdot\mu)}{(1-\eta-\eta\cdot\mu)}\cdot\frac{\bar{e}}{e}\cdot w-\theta\right]\right\}, \tag{14}$$

where ε is credit supply elasticity with respect to wealth (owned farm size). In the special case $\mu=0$ (no effect of family supervision on hired laborers' effort) and given the assumption $w\geqq\theta$, the share of operated area relative to owned area will be declining provided credit elasticity is not much more than unity (i.e., larger farmers are only slightly more than proportionately favored by credit suppliers). In the more general case $\mu>0$, the share of farm area operated by the household may increase with owned farm size if η is less than $\frac{1}{2}$ and $\varepsilon>1$. However, if $\eta\geqq\frac{1}{2}$ and ε is not much more than unity, the share of land operated by the household will clearly decline with owned farm size. Eq. (14) is of special interest in the case where no rental market exists, since in that case (with $R=0$) it describes the pattern of land utilization. The discussion of this case is deferred to a later section.

[8]The full term is $\mathrm{d}Q/\mathrm{d}A=[1-\eta-\eta\cdot\mu+\eta\cdot\mu\cdot(F/L)]\cdot(Q/A)$.

To demonstrate that the relation between per-acre yields and operational holding size can follow different patterns within the framework of the present model, we use the definition of effective labor and the first order conditions to calculate the *optimal* per-acre input of labor.

$$(L/A)^* = \eta\cdot\{[(c+R)\cdot e/w]-[\mu\cdot\bar{e}\cdot F/A]\}/(1-\eta-\eta\cdot\mu). \tag{15}$$

Differentiation of eq. (15) with respect to owned holding size V yields[9]

$$\frac{\mathrm{d}(L/A)^*}{\mathrm{d}V} = \mu\cdot\eta\cdot\left[\frac{\bar{e}}{e}\cdot\frac{F}{A}-\frac{(c+R)}{w}\right]\cdot\frac{e}{A}\cdot\frac{\mathrm{d}A}{\mathrm{d}V}\bigg/(1-\eta-\eta\cdot\mu). \tag{16}$$

Inspection of eq. (13) verifies that $[\mathrm{d}A/\mathrm{d}V]/(1-\eta-\eta\cdot\mu)>0$ and the sign of eq. (16) thus depends on the term in square brackets. It can be easily shown that when $1-\eta-\eta\cdot\mu<0$, it must hold $\mathrm{d}(L/A)/\mathrm{d}V>0$, since in that case larger owners operate smaller farms and spend more per acre on hired labor. This must also hold in the case $1-\eta-\eta\cdot\mu=0$, since in that case operational farm size is independent of wealth, but the per-acre spending on hired labor increases with wealth. In the case $1-\eta-\eta\cdot\mu>0$, the relation between the effective labor input per acre and owned holding size can be negative or positive. Consider, for instance, the case $\eta=\frac{1}{2}$. First order conditions imply $(1-\eta)(\bar{e}/e)\cdot(F/A)-\eta\cdot((c+R)/w)<0$, hence, in the case $\eta=\frac{1}{2}$, it follows $\mathrm{d}(L/A)/\mathrm{d}V<0$, i.e., the effective labor input declines with owned holding size. The same result can be obtained for all $\eta<\frac{1}{2}$. By an argument of continuity, since in the case $1-\eta-\eta\cdot\mu=0$ it holds $\mathrm{d}(L/A)/\mathrm{d}V>0$, there must exist some low (but positive) values of the term $(1-\eta-\eta\cdot\mu)$ for which $\mathrm{d}(L/A)/\mathrm{d}V>0$ holds. The relation between per-acre yields and owned-holding size follows the same pattern as the per-acre labor input, and the conclusion is, therefore, that one may observe a positive or a negative relation between *operational* holding size and per-acre yields, depending on the relative magnitudes of η and μ. In the case $1-\eta-\eta\cdot\mu=0$ there will be no correlation between operational holding size and per-acre yields. These results are compatible with the existence of conflicting empirical evidence on the nature of the relation between these variables in different areas [Berry and Cline (1979, p. 225, fn. 21), Deolalikar (1981, p. 275)]. A model where labor effectiveness is not affected by supervision ($\mu=0$) would predict $\mathrm{d}(L/A)/\mathrm{d}V=0$, and cannot therefore provide an explanation to various patterns observed, unless other elements (e.g., differential prices, land quality) are introduced.

It can be shown that the number of hired laborers per operated acre increases with owned holding size. It follows therefore that in the case $1-\eta-$

[9]Eq. (15) and the subsequent derivations assume $1-\eta-\eta\cdot\mu\neq 0$. The special case $1-\eta-\eta\cdot\mu=0$ will be discussed separately.

$\eta \cdot \mu < 0$ the number of hired laborers per acre is negatively related to operational holding size, while in the case $1 - \eta - \eta \cdot \mu > 0$ the relation between the two variables is positive.[10] In the latter case there will also be a negative relation between the share of hired labor and per-acre output.[11]

So far, the analysis in this section assumed that the number of family members is given. However, the labor input and operational farm size are affected by the size of the family. It can be shown that the operational holding is related to the family size by the following condition:

$$\operatorname{sign}(\mathrm{d}A/\mathrm{d}F) = \operatorname{sign}\left\{(1-\eta)\cdot(1-\mu)\frac{\bar{e}}{e} - (1-\eta-\eta\cdot\mu)\cdot\frac{\theta}{w}\right\}. \tag{17}$$

Clearly, in the case $1-\eta-\eta\cdot\mu \leqq 0$ larger families maintain larger operational holdings. This also holds in the case $1-\eta-\eta\cdot\mu>0$ provided $\eta \leqq \frac{1}{2}$, but more generally it is possible that larger families will maintain smaller holdings.

It is intuitively expected (and can indeed be verified) that the number of hired laborers per acre declines as the family size increases. The relation between the effective labor input and yield per acre is governed by a condition opposite in sign to that of $\mathrm{d}(L/A)/\mathrm{d}V$, i.e.,

$$\operatorname{sign}[\mathrm{d}(L/A)/\mathrm{d}F] = -\operatorname{sign}[\mathrm{d}(L/A)/\mathrm{d}V]. \tag{18}$$

An issue ignored so far is the possibility of hiring out family labor. It can be shown that, as long as the family employs hired labor, it is not optimal to release a family member for off-farm employment,[12] unless he can secure a salary higher than the going agricultural wage rate w. However, since we have established $\mathrm{d}N/\mathrm{d}F<0$ and $\mathrm{d}N/\mathrm{d}V<0$, it is clear that families with sufficiently small owned holdings and/or sufficiently large family sizes will not require hired labor. At this point the possibility of off-farm employment is relevant. Technically, the analysis in this case is similar to the special case $\mu=0$ and the same first order conditions for optimality hold. The comparative static and comparative dynamics results to be derived in subsequent sections for the case $\mu=0$ are valid for the case of a family farm with no hired labor.

4. Impact of parameter changes

Several comparative static results are presented in table 1. These results

[10]See Brown and Salkin (1974, p. 152) for evidence on such a relationship.

[11]See Bhalla (1979, p. 145).

[12]We are abstracting from the possibility that family labor can be offered for off-farm employment in the slack season.

Table 1
Comparative static results.[a]

Variable affected	Parameter changed: c	w	R	s^{b}
A (desired operational holding)	$-$	$+$	$-$	$-$ / 0 / $+$ if $1-\eta-\eta\cdot\mu \gtreqless 0$
$A\cdot N$ (hired labor employment per farm)	$+$ if $\mu>0$; 0 if $\mu\leq 0$	$-$	$+$	$+$

[a]Proofs of the signs are available from the author upon request.
[b]Assuming the supply of credit is proportional to owned farm size, i.e., $S(V)=s\cdot V$

are of a partial equilibrium nature since they are derived under the assumption that other prices remain unaffected.

A reduction in intermediate input cost (c) will increase the desired operational farm size, but will reduce hired labor employment per farm if rental rates remain unchanged, provided $\mu>0$. If supervision does not affect hired labor effort, employment per farm is unchanged.[13]

An increase in the availability of credit (higher per-acre owned credit supply) may reduce or increase the desired operational farm size, depending on the sign of the term $1-\eta-\eta\cdot\mu$. The employment of hired labor will increase.

The demands for hired labor and land are negatively related to the respective prices (wage rate and rental rate), as one would expect. The substitution effects are positive, i.e., higher wage rates increase the desired operational farm size while higher rental rates increase the demand for hired labor.

5. The case with no rental market

While the model developed in the preceding sections assumes that land can be rented in or leased out freely, there is a certain fixed cost involved which may inhibit or even eliminate rental transactions for some households. This cost is incurred due to the fact that not all additional tracts of land available for rental are located close to the holding already operated. When an incorporation of an additional tract in the operational holding necessitates time consuming shuttles, the added cost may wipe out the additional profit that an additional contiguous tract could provide. Fragmentation also

[13]This is strictly correct only in a partial equilibrium context. As shown in a subsequent section rental rates will increase, thus offsetting at least some of the initial effects of a reduction in intermediate input costs.

reduces the effectiveness of supervision, given the size of the family. Indeed, in areas where population pressure over generations tended to produce fragmentation of holdings, one observes a continuous effort by farmers to consolidate holdings by buying and selling land or by exchanging land.

Extension of the analysis to the case where no rentals are feasible can be accomplished with very few changes. Considering first the case where credit constraints are not binding, the earlier analysis is still applicable with $R=0$ for those households whose optimal holding is lower than their owned land. The difference $V-A$ is left unutilized. For households who would like to operate more land than they own, the optimization problem involves one control variable only, namely, the number of hired laborers (while the operated land is restricted to equal V). The optimal solution is characterized by eq. (8) with $\lambda=0$. A straightforward differentiation establishes that, while the relation between farm size and the number of hired laborers per acre is ambiguous, the effective labor input (and yield) declines with farm size (provided the size of the owned holding is an effective constraint on the desired operational holding size). This result holds only for the case $\mu>0$. If, however, labor effort is not affected by supervision ($\mu=0$), the optimal effective labor input is identical on all farms, and yield is not affected by farm size.

Since for larger farms the absence of rental possibilities implies unutilized land, and given that the optimal operational holding for these farms is unaffected by size, it follows that the rate of utilization declines with farm size [evidence on such a pattern is discussed in Berry and Cline (1979)].

With a binding credit constraint, owned farm size may not be an effective upper limit on the desired operational holding for the larger farms. The earlier comparative static results are thus valid for the larger farms, with $R=0$. For the smaller farms, the operational holding coincides with the owned holding and the hired labor force is determined by the cash constraint. It can be shown that $S' \geqq S/V$ is a sufficient (but not necessary) condition for $\mathrm{d}N/\mathrm{d}V>0$, and ($S' \leqq S/V$, $\theta \leqq w$) are sufficient (but not necessary) conditions for $\mathrm{d}(L/A)/\mathrm{d}V<0$, provided $\mu>0$. If $\mu=0$, the latter conditions become necessary and sufficient. It follows that when labor effort is responsive to supervision and if a rental market does not exist, a negative relation between farm size and land productivity is more likely. As in the case without credit constraints, the smaller farmers utilize all of their land while larger land owners leave some land unutilized. Insights regarding the rate of land utilization can be gained from eq. (14), under the assumption $R=0$. The often observed negative relation between land utilization and owned farm size is clearly possible if the availability of credit does not increase more than proportionately with owned farm size and η is large. If however, η is small (implying that partial output elasticity with respect to land is large), and if also μ is large, it is possible that land utilization will increase with owned farm size. The likelihood of a negative relation between land utilization and farm size declines if credit supply favors significantly larger landowners (ε much larger than 1), but such supply conditions do not eliminate the possibility of a negative relation. Berry and Cline (1979) observed for a number of countries a negative relation between output per available (rather than cultivated) acre and holding size. In the present case, a differentiation of Q/V yields (utilizing the first order conditions)

$$\frac{\mathrm{d}(Q/V)}{\mathrm{d}V}=\frac{\varepsilon\cdot(1-\eta)\cdot S-\mu\cdot(S-\theta\cdot F)-(1-\mu)\cdot c\cdot A}{[c+\mu\cdot N/w]/q}. \tag{19}$$

The sign of eq. (19) can be positive, negative, or zero, and it can change signs as well.

6. General equilibrium under a binding credit constraint

While the wage rate may be considered exogenous, assuming that a large pool of landless labor is available, the rental rate for land is endogenously determined in the model.[14] Denote the joint frequency function of owned housing sizes and family sizes by $G(V,F)$. The individual demand for rentals is given by $[A(V,F,R)-V]$. The equilibrium condition requires that excess demand for rentals be zero, i.e.,

$$\int_F\int_V [A(V,F,R^*)-V]\cdot G(V,F)\cdot \mathrm{d}V\cdot \mathrm{d}F=0, \tag{20}$$

where R^* denotes the equilibrium rental rate. Since we have already shown $\mathrm{d}A/\mathrm{d}R<0$ (table 1), it follows that the excess demand curve is negatively sloped. This ensures a unique and stable equilibrium.

From the results in table 1, it follows that a subsidy reducing the cost of non-labor inputs will increase the equilibrium rental rate. Similarly, a minimum wage legislation which increases the wage rate will increase the equilibrium rental rate.

Changes in the supply of credit may increase or reduce the equilibrium rental rate depending on the sign of $(1-\eta-\eta\cdot\mu)$.

While an explicit solution of the equilibrium rental rate is difficult to obtain in general, the special case $1-\eta-\eta\cdot\mu=0$ is tractable and yields the equilibrium rate

$$R^*=[w\cdot\bar{e}\cdot(1-\eta)/\eta]\cdot(\bar{V}/\bar{F})^{\mu-1}-c, \tag{21}$$

[14]This, of course, is not the most general case. Rosenzweig (1978) considers a general equilibrium model of the rural economy where there is no rental or land market, but the wage rate is endogenous. The general equilibrium results of the present model may, therefore, be viewed as valid only for situations where the supply elasticity of labor is high.

where $\bar{V}$ and $\bar{F}$ are the average owned holding size and average family size, respectively. It follows therefore that the higher the land/population ratio (where population refers to land owners only), the lower the equilibrium rental rate. This special case can be used to demonstrate that the partial equilibrium effect of a policy may be quite different from its general equilibrium effects. It was already pointed out that a subsidy on the price of non-labor inputs will tend to reduce employment while operational holdings expand. It is obvious from eq. (21), however, that the sum R^*+c remains constant (i.e., the rental rate will increase by exactly the amount of cost reduction), and therefore, by the first order conditions, the general equilibrium size of operational holdings remains unchanged. With A and $c+R$ unchanged, it follows from eq. (9a) that the number of hired employees per operated acre increases, and thus, the general equilibrium outcome of the subsidy is increased employment.

7. A numerical example

Some of the quantitative implications of the model incorporating the supervision role of family members are illustrated in the numerical example constructed below. The underlying assumptions are: (i) no rental market ($R=0$, $A \leqq V$), (ii) fixed elasticities in the functions q and e [i.e., $q=(L/A)^{\eta}$ and $e=(F/A)^{\mu}$], (iii) credit supply is proportional to owned land (i.e., $S=s\cdot V$).

With these specifications, the first order conditions [eqs. (7)–(9)] can be manipulated to yield the following relation (when $A>0$, $N>0$, $\lambda>0$):

$$(1-\eta\cdot\mu)\cdot(c/w)\cdot A-(1-\eta)\cdot\bar{e}\cdot F^{1-\mu}\cdot A^{\mu}-(1-\eta-\eta\cdot\mu)\cdot(s\cdot V-\theta\cdot F)/w=0. \tag{22}$$

This polynomial yields explicit solutions for A (the desired operational farm size) in the two special cases $\mu=0$ and $\mu=\frac{1}{2}$, which will be used to compare results.[15]

Once solutions for A are obtained for various values of V (owned land), land utilization rates can be calculated. Effective per-acre labor inputs can be computed using eq. (15), and these are in turn utilized to calculate per-acre yields.

[15]The solutions are

$$A(\mu=0)=(1-\eta)\cdot[(w-\theta)\cdot F+s\cdot V]/c,$$

$$A(\mu=\tfrac{1}{2})=\left\{\frac{(1-\eta)\cdot\bar{e}\cdot F^{0.5}+\sqrt{(1-\eta)^2\cdot\bar{e}^2\cdot F+4\cdot[(1-0.5\eta)\cdot c]\cdot(1-1.5\eta)\cdot(s\cdot V-\theta F)/w^2}}{[2\cdot(1-0.5\eta)\cdot c/w]}\right\}^2.$$

The various parameters in eq. (22) are assigned the following values:

$$w=\theta=1,\quad F=4,\quad \bar{e}=1,$$

$$c=1,\quad s=2.222\ldots,\quad \eta=0.55.$$

It should be noted that these values are compatible with data from LDCs. Kutcher and Scandizzo (1981, pp. 94–95) estimated values of production elasticities with respect to labor (η) for Northeast Brazil in the range 0.51–0.58. The data from the Muda region in Malaysia [Bell et al. (1982, pp. 33, 39)] indicate that the ratio of per-acre production costs (excluding labor and rent) to average per family member consumption was 1.44 while in the present case $c/\theta=1$. The same data suggest that per-acre production costs are about 20% of per-acre output value. If this is applied in the present example, then the value assumed for s would imply that the credit limit per acre is half of the per-acre production value, which is reasonable for a short-term loan.

As is apparent from table 2, the assumed parameter values imply for the case where family supervision has no effect on labor effectiveness ($\mu=0$) that all owned land is utilized (utilization rate 1), irrespective of the size of the farm. The case $\mu=\frac{1}{2}$, on the other hand, generates a declining utilization rate

Table 2
Results of the numerical example.

Owned land (1) V	Desired operated land (A) (2) $\mu=0$	$\mu=\frac{1}{2}$	Land utilization ratio (A/V) (3) $\mu=0$	$\mu=\frac{1}{2}$	Per-acre effective labor input L/A (4) $\mu=0$	$\mu=\frac{1}{2}$	Per-acre yield index[a] q (5) $\mu=0$	$\mu=\frac{1}{2}$	Berry and Cline data on land utilization rates in Colombia[b] (6)
5	5	4.29	1	0.86	1.22	1.57	100	100	0.84
10	10	7.88	1	0.79	1.22	1.44	100	95.4	0.82
20	20	14.48	1	0.72	1.22	1.22	100	90.0	0.78
30	30	20.79	1	0.69	1.22	1.07	100	81.3	0.70
50	50	32.98	1	0.66	1.22	0.90	100	73.8	0.70
100	100	62.49	1	0.62	1.22	0.69	100	63.9	0.58
500	500	288.31	1	0.58	1.22	0.35	100	43.7	0.50

[a]The index assumes yield level at $V=5$ as 100.
[b]Berry and Cline [1959, p. 61, table 4.9, column (6)].

as owned farm size increases [column (3) of table 2]. These utilization rates are quite compatible with data from Colombia reported in Berry and Cline (1979), which are presented in column (6) of table 2. The effective labor input does not vary with farm size in the case $\mu=0$, as predicted by eq. (16). In the case with family supervision effect ($\mu=\frac{1}{2}$), effective labor input per acre declines with owned farm size. The per-acre yield in the no-supervision effect ($\mu=0$) is constant, while in the case $\mu=\frac{1}{2}$ the yield for the small farm size is about 33% higher than that of the medium farm size and double that of the large farm size. These results are broadly compatible with data on yields by farmers of different farm size classes in several LDCs reported in Berry and Cline.

8. Summary and concluding remarks

This paper has shown that the role of family members as supervisors of hired labor, and the positive relation between employees' productivity and supervision intensity, provide a possible explanation to the variety of patterns and 'stylized facts' observed in the relation between farm sizes, input use and average output levels. While family manpower is a fixed resource on each farm, and even though its supervisory capacity cannot be traded, market forces would generate an optimal solution (from society's point of view) if capital and land rental markets were perfect and if each household maximized its profit. This solution follows from the fact that with perfect markets, each family leases in or leases out as much as is required to maintain an optimal operational holding which is proportionate to size of the family. Labor inputs per acre are identical across farms and thus yields are unaffected by farm size.

The results are quite different when an imperfect credit market is assumed, whereby the amount of working capital available to each household is determined according to the amount of collateral (owned land) it can offer. When credit supply limits the amount of cash outlays the household can undertake, the pattern of land holding and resource utilization depends on the relative magnitudes of output elasticity with respect to effective labor[16] and labor effort elasticity with respect to supervision. Even if land rental markets are perfect and transaction costs are ignored, the optimal operational holding size for each household will vary systematically with owned holding size. The higher is labor elasticity with respect to supervision, and the lower is output elasticity with respect to land, the weaker is the positive relation between owned land and the optimal operational holding, and with extreme values of the above mentioned elasticities this relation may even be reversed. The yield per acre of cultivated area, which would be identical

[16]Given the assumption of constant returns to scale, partial output elasticity with respect to effective labor is one minus partial output elasticity with respect to land.

across all farm sizes in the case that labor effectiveness is not enhanced by supervision, could be increasing with farm size if output elasticity with respect to land is low and labor effort elasticity with respect to supervision is high. However, if the value of output elasticity with respect to land is high the yield per acre may be declining with owned farm size.

Investigation of the impact of price and credit supply changes within the framework of the credit constrained model shows that a subsidy on non-labor inputs will tend to reduce total employment in the short run, if labor effort is responsive to supervision, but will have no effect on employment in the absence of supervision effect. In a general equilibrium context, however, such a policy is likely to increase rents, and since higher rents cause an increase in employment, the initial employment reduction effect may be modified or may even be reversed (as shown for one special case). Higher wage rates tend to increase the size of operational holdings and reduce employment. This, in turn, will increase rental rates and therefore weaken somewhat the initial employment impact. An increase in credit supply may reduce or increase desired operational holdings, but its overall impact on employment is favorable in the short run and possibly also in the long run.

All the results reported above are obtained without necessitating assumptions regarding farm size related price distortions (e.g., differential prices for large and small farmers). The present model does assume a size dependent supply of credit, but the per-acre supply of credit can be neutral to scale without affecting model results. Price distortions are usually suggested as the source of observed differential patterns of resource utilization across farm size groups. The present model thus not only provides an additional plausible explanation, but will also hold in the absence of price distortions. It also follows that policies to remove price distortions may not eliminate the differential patterns of resource utilization if credit availability depends on farm size.

References

Bardhan, Pranab, 1973, Size productivity and returns to scale: An analysis of farm level data in Indian agriculture, Journal of Political Economy 81, no. 6, 1370–1386.

Bell, Clive, Peter Hazell and Roger Slade, 1982, Project evaluation in regional perspective (World Bank and Johns Hopkins University Press, Baltimore, MD).

Berry, R. Albert and William R. Cline, 1979, Agrarian structure and productivity in developing countries (Johns Hopkins University Press, Baltimore, MD).

Bhalla, Surjit, 1979, Farm size, productivity, and technical change in Indian agriculture, Appendix A, in: Berry and Cline (1979) 141–193.

Binswanger, Hans P. and Mark R. Rosenzweig, 1983, Behavior and material determinants of production relations in agriculture, Report no. ARU 5 (Research Unit, Agriculture and Rural Development Department, World Bank, Washington, DC).

Brown, James A. and Jay S. Salkin, 1974, Underemployment in rural South Vietnam: A comment and a discussion of family labor, Economic Development and Cultural Change 23, no. 1, 151–160.

Deolalikar, Anil B., 1981, The inverse relationship between productivity and farm size: A test using regional data from India, American Journal of Agricultural Economics 63, no. 2, 275–279.

Deolalikar, Anil B. and Wim P.M. Vijverberg, 1982, The heterogeneity of family and hired labor in agricultural production: A test using district level data from India, Discussion paper no. 411 (Economic Growth Center, Yale University, New Haven, CT).

Desai, Meghnad and Dipak Mazumdar, 1970, A test of the hypothesis of disguised unemployment, Economica xxxvii, no. 145, 39–53.

Kutcher, Gary and Pasquale Scandizzo, 1981, The agricultural economy of North-East Brazil (World Bank and Johns Hopkins University Press, Baltimore, MD).

Rao, Vaman and Tosporn Chotigeat, 1981, The inverse relationship between size of land holding and agricultural productivity, American Journal of Agricultural Economics 63, no. 3, 571–574.

Rosenzweig, Mark R., 1978, Rural wages, labor supply and land reform, American Economic Review 68, no. 5, 847–861.

Rosenzweig, Mark R., 1980, Neoclassical theory and the optimizing peasant: An econometric analysis of market family labor supply in a developing country, Quarterly Journal of Economics XCIV, no. 1, 31–56.

Smith, Vernon, 1972, A theory and test of credit rationing. Some generalizations, American Economic Review LXII, no. 3, 477–483.

Squire, Lyn, 1981, Employment policy in developing countries (Oxford University Press, New York).