Uzawa's Theorem

Background

Unlike the prior projects, this one does not deal with a specific empirical relationship, but rather with a particular theoretical finding. Uzawa (1961), provided a proof that is often called the "Steady-State Growth Theorem". To state the theorem, start by defining clearly what a "balanced growth path" (BGP) entails:

1. The growth rate of output per worker, consumption per worker, and capital per worker are equal, and constant over time
2. The factor shares of capital and labor are constant over time
3. The rate of return on capital is constant over time

Next, let output be determined by $Y_t = F(B_t K_t, A_t L_t)$, which has the following properties

1. $F()$ is constant returns to scale with respect to the two arguments
2. B_t is "capital-augmenting" technological change
3. A_t is "labor-augmenting" technological change, often called "Harrod-neutral change"

The Steady-State Growth Theorem says that if an economy has a BGP with a growth rate that is greater than zero, then (a) it must be that this growth rate is equal to $\dot{A}/A$ and (b) it must be that $\dot{B}/B = 0$. In other words, a BGP requires technological change to be labor-augmenting, and labor-augmenting *only*.

Project

Prove it.

Rules

You can work on this alone, or with a small group (2-3 people). I'll evaluate the work of the group as a whole.